Hanaki, A. and Hida, A.
Osaka J. Math.
29 (1992), 71–74

A REMARK ON M_p-GROUPS

Dedicated to Professor Kazuhiko Hirata on his 60th birthday

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(Received November 1, 1990)

1. Introduction

Let FG be the group algebra of a finite group G over an algebraically closed field F of characteristic $p>0$. We call an FG-module V monomial if V is induced from some 1-dimensional FH-module for some subgroup H of G. An ordinary character χ of G is called monomial if χ is induced from some linear character of some subgroup of G. We call G an M_p-group if every irreducible FG-module is monomial. We call G an M-group if every irreducible ordinary character of G is monomial. For details, see a paper of Bessenrodt [1] and a book of Isaacs [4]. It is well known that M-groups are solvable (15.7 in [2]). M_p-groups are also solvable (3.8 in [6]). By Fong-Swan's theorem, M-groups are M_p-groups for any prime p. But M_p-groups need not be M-groups. For example, $SL(2, 3)$ is an M_2-group but not an M-group. So we investigate conditions for M_p-groups to be M-groups. Namely,

Theorem 3. *Let G be a p-nilpotent group. Then G is an M-group if and only if G is an M_p-group.*

Throughout this paper, groups are finite groups, F is an algebraically closed field of characteristic $p>0$, FG-modules are finitely generated right FG-modules, and characters are ordinary characters. Let χ be a character of a group G. We write χ^* for the Brauer character corresponding to χ. Let H be a subgroup of G and φ be a character of H. We write χ_H for the restriction of χ to H and φ^G for the induced character from φ. We use the same notation for modules. When M and N are FG-modules, we write $N\,|\,M$ if N is a direct summand of M. We write $\mathrm{Irr}(G)$ for the set of all irreducible characters of G. For the other notation and terminology we shall refer to books of Dornhoff [2] and Nagao and Tsushima [5].

We wish to thank S. Koshitani for many helpful conversations during the course of this work.