Sankaran, G.K.
Osaka J. Math.
29 (1992), 63-70

REMARKS ON OPEN SURFACES

G.K. SANKARAN

(Received February 8, 1991)

0. Introduction

Let X be a smooth complex analytic open surface: that is, X is biholomorphically equivalent to $M\setminus D$, where M is a compact complex variety of dimension 2 and D is a closed analytic subvariety of M. We can assume that M is also smooth.

We shall be interested in the case where X is strongly pseudoconvex (see [6] for definitions). Such an X contains a distinguished compact analytic subset Z, which is the union of all closed analytic subspaces of X of positive dimension. Z is empty if and only if X is Stein.

A famous remark of Serre, in [8], points out that M is not determined up to bimeromorphic equivalence by X. If $X=\boldsymbol{C}^*\times\boldsymbol{C}^*$ then M can be rational or elliptic ruled, or (an observation due to Igusa, see [1]) a non-elliptic Hopf surface. Naturally one asks: for what other such X, if any, is M not unique up to bimeromorphic equivalence?

This question and some related ones have been considered by (among others) Tan, in a series of papers ([11], [12], [13], [14]). There it is always assumed that M is minimal. This, however, imposes a further restriction on X. The purpose of this note is to see what happens for general M.

1. A non-minimal example

We give an easy example of a Stein open surface X for which only non-minimal compactifications exist. Let M' be a surface whose universal cover is a bounded domain, say a ball in $\boldsymbol{C}^2$. Then M' is a strongly minimal surface of general type and is hyperbolic in the sense of [4]. Let $\pi: M\to M'$ be the blow-up of M' in a point p, and let $C=\pi^{-1}(p)$ (so that C is a (-1)-curve in M). Fix some projective embedding of M and let D be a general hyperplane section (so $C\nsubseteq D$). Put $X=M\setminus D$: thus X is affine, and therefore Stein. As we shall see below, the fact that M is of general type implies that it is determined by X up to bimeromorphic equivalence. By the uniqueness of minimal models, M' is the only minimal surface which can possibly contain an open subset biholomorphically equivalent to X. Suppose it does: let $\varphi: X\hookrightarrow M'$ be a