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E_*-INJECTIVE SPECTRA AND INJECTIVE E_*E-COMODULES

Dedicated to Professor Haruo Suzuki on his sixtieth birthday

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0. Introduction

In [13] Ohkawa introduced the notion of the injective hull of spaces and spectra with respect to homology and proved the existence theorem [13, Theorem 1]. Following [13, Definition 1 i)] we call a CW-spectrum W *E_*-injective* if any map $f: X \to Y$ induces an epimorphism $f^*: [Y, W] \to [X, W]$ whenever $f_*: E_*X \to E_*Y$ is a monomorphism, for a fixed CW-spectrum E. A CW-spectrum W is E_*-injective if and only if the homomorphism $\kappa_E: [X, W] \to \mathrm{Hom}(E_*X, E_*W)$ defined by $\kappa_E(f)=f_*$ is a monomorphism for any CW-spectrum X (see [13, Proposition 7]). In this note we will be concerned about E_*-injective spectra.

For each CW-spectrum X, E_*X is regarded as a module over the algebra E^*E of cohomology operations. Under the restriction that E is finite, Ohkawa [13, Theorem 3 i) and iii)] gave the following characterization.

Theorem 0. *Assume that a CW-spectrum E is finite. Then the following conditions are equivalent:*

i) *W is an E_*-injective spectrum.*

ii) *W is an E_*-local spectrum such that E_*W is injective as an E^*E-module.*

iii) *$\kappa_E: [X, W] \to \mathrm{Hom}_{E^*E}(E_*X, E_*W)$ is an isomorphism for any CW-spectrum X.*

According to [2, Proposition III.13.4] (or see [1]), the well known ring spectra $E=S$, HZ/p, MO, MU, MSp, KU and KO satisfy some of nice properties as stated in the beginning of §2. For example, E_*E becomes flat as an E_*-module, and then E_*X may be regarded as a comodule over the coalgebra E_*E. In §2 we will prove the following result (Theorem 2.1) for such a nice ring spectrum E, corresponding to Theorem 0 for a finite spectrum E.

Theorem 1. *Let E be a ring spectrum such that E_*E is flat as an E_*-module. Assume that E satisfies the property* (K″) *stated in the beginning of* §2. *Then the following conditions are equivalent:*