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HEEGAARD SPLITTING FOR SUTURED MANIFOLDS AND MURASUGI SUM

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1. Introduction

The concept of sutured manifolds was introduced by Gabai, and it has been playing an important role in the 3-manifold theory ([3]-[7]). On the other hand, in [2], Casson and Gordon defined Heegaard splittings of compact orientable 3-manifolds with boundaries by using compression bodies. We note that this enables us to define Heegaard splittings for sutured manifolds. In this paper, we study complementary sutured manifolds for Seifert surfaces from the viewpoint of this Heegaard splitting.

Firstly, we give the definition of Heegaard splittings for sutured manifolds following [2]. A *compression body* W is a cobordism rel ∂ between surfaces $\partial_+ W$ and $\partial_- W$ such that $W \cong \partial_+ W \times I \cup$ 2-handles $\cup$ 3-handles and $\partial_- W$ has no 2-sphere components. It is easy to see that if $\partial_- W \neq \phi$ and W is connected, W is obtained from $\partial_- W \times I$ by attaching a number of 1-handles along the disks on $\partial_- W \times \{1\}$ where $\partial_- W$ corresponds to $\partial_- W \times \{0\}$. We denote the number of these 1-handles by $h(W)$. Let (M, γ) be a sutured manifold such that $R_+(\gamma) \cup R_-(\gamma)$ has no 2-sphere components and $T(\gamma)=\phi$. We say that (W, W') is a *Heegaard splitting* of (M, γ) if both W and W' are compression bodies, $M=W \cup W'$ with $W \cap W'=\partial_+ W=\partial_+ W'$, $\partial_- W=R_+(\gamma)$, and $\partial_- W'=R_-(\gamma)$. Assume that $R_+(\gamma)$ is homeormorphic to $R_-(\gamma)$. Then we define the *handle number* $h(M, \gamma)$ of (M, γ) as follows:

$$h(M, \gamma) = \min \{h(W); (W, W') \text{ is a Heegaard splitting of } (M, \gamma)\}.$$

Note that $h(M, \gamma)$ corresponds to the Heegaard genus of a closed 3-manifold.

For the definitions of a $2n$-Murasugi sum and a complementary sutured manifold, see Section 2. Let R be a Seifert surface in S^3 obtained by a $2n$-Murasugi sum of two Seifert surfaces R_1 and R_2 whose complementary sutured manifolds (M_i, γ_i) $(i=1, 2)$ are irreducible. Let (M, γ) be the complementary sutured manifold for R. In this paper, we consider the relations between $h(M_i, \gamma_i)$ $(i=1, 2)$ and $h(M, \gamma)$. In fact, we prove: