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PRO-l PURE BRAID GROUPS OF RIEMANN SURFACES AND GALOIS REPRESENTATIONS[1]

To the memory of the late Professor Michio Kuga

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Introduction

Let X be a smooth irreducible algebraic curve of genus g over a field k of characteristic 0, and l be a prime number. For each $n=1, 2, \cdots$, consider the configuration space

$$Y_n = F_{0,n}X = \{(p_1, \cdots, p_n) \in X^n; p_i \neq p_j \quad \text{for} \quad i \neq j\} .$$

Then the Galois group $\mathrm{Gal}(\bar{k}/k)$ acts outerly on the pro-l fundamental group $P_n = \pi_1^{pro-l}(Y_n)$;

$$\varphi_n \colon \mathrm{Gal}(\bar{k}/k) \to \mathrm{Out}\, P_n .$$

The main purpose of this paper is to prove that φ_n has the same kernel for all sufficiently large $n \geq n_0 = n_0(X/k, l)$ (Theorem 2, §4). For example, we can take $n_0=1$ if $g \geq 1$ and X is affine, $n_0=2$ if $g \geq 1$, and $n_0=4$ in all cases. This theorem is based on some group theoretic property of Out P_n (Theorem 1, §1).

The present work grew out of our previous work [7], [8] and [6].

1. The statement of Theorem 1

1.1. Let X^{cpt} be a compact Riemann surface of genus $g \geq 0$, and $X = X^{cpt} \setminus \{a_1, \cdots, a_r\}$ $(r \geq 0)$ be the complement of r distinct points $a_1, \cdots, a_r$ in X^{cpt}. For each integer $n \geq 1$, consider the configuration space

$$Y_n = F_{0,n}X = \{(p_1, \cdots, p_n) \in X^n; p_i \neq p_j \quad \text{for} \quad i \neq j\} ,$$

and let $\pi_1(Y_n, b)$ be its fundamental group with a base point $b=(b_1, \cdots, b_n)$. It is the pure braid group of X with n strands. For each i $(1 \leq i \leq n, n \geq 2)$, the projection

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