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INVARIANT DIFFERENTIAL OPERATORS ON THE GRASSMANN MANIFOLD $SG_{2,n-1}(R)$

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0. Introduction. The Grassmann manifold $\boldsymbol{SG}_{2,n-1}(\boldsymbol{R})=SO(n+1)/SO(n-1)\times SO(2)$ with its canonical Riemannian metric is known to be a Riemannian symmetric space of rank 2. Hence the algebra $\boldsymbol{D}(\boldsymbol{SG}_{2,n-1}(\boldsymbol{R}))$ of $SO(n+1)$-invariant (linear) differential operators on $\boldsymbol{SG}_{2,n-1}(\boldsymbol{R})$ is generated by two differential operators.

It is the aim of our paper to exhibit simultaneous eigenspace decomposition of appropriate generators $\Delta_0^\wedge$ and $\Delta_1^\wedge$ of the algebra $\boldsymbol{D}(\boldsymbol{SG}_{2n-1}(\boldsymbol{R}))$. We have obtained in [7] the followings:

(1) the eigenspace decomposition of Δ_0 restricted to $\boldsymbol{K}^*(S^n, g_0)$ is given, where g_0 is the canonical metric on S^n and Δ_0 is the Lichnerowicz operator acting on the graded algebra $\boldsymbol{S}^*(S^n, g_0)$ of symmetric tensor fields on the standard sphere (S^n, g_0) and $\boldsymbol{K}^*(S^n, g_0)$ is the graded subalgebra of $\boldsymbol{S}^*(S^n, g_0)$ generated by Killing vector fields,

(2) Radon transform $\wedge$:

$$\boldsymbol{S}^*(S^n, g_0)\rightarrow C^\infty(\boldsymbol{SG}_{2,n-1}(\boldsymbol{R}))$$

intertwines Δ_0 with the Laplace Beltrami operator $\Delta_0^\wedge$ on $\boldsymbol{SG}_{2,n-1}(\boldsymbol{R})$, i.e.,

$$(\Delta_0\xi)^\wedge = \Delta_0^\wedge \xi^\wedge$$

for $\xi\in\boldsymbol{S}^*(S^n, g_0)$,

(3) the eigenspace decomposition obtained in (1) is transferred to that of $\Delta_0^\wedge$, since the kernel of the Radon transform restricted to $\boldsymbol{K}^*(S^n, g_0)$ is the principal ideal generated by $g_0/2-1$ and the image of $\boldsymbol{K}^*(S^n, g_0)$ is uniformly dense in $C^\infty(\boldsymbol{SG}_{2,n-1}(\boldsymbol{R}))$.

In the present paper a new differential operator Δ_1 which acts on $\boldsymbol{S}^*(S^n, g_0)$ with analogous properties as (1), (2), (3) above is constructed.

Especially $\Delta_0^\wedge$ together with the differential operator $\Delta_1^\wedge$ corresponding to Δ_1 by the Radon transform are found to be a set of generators of the algebra $\boldsymbol{D}(\boldsymbol{SG}_{2,n-1}(\boldsymbol{R}))$.

In **1** and **2**, we recall the results obtained in [7] with some improvements.