Mall, D.
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THE COHOMOLOGY OF LINE BUNDLES ON HOPF MANIFOLDS

DANIEL MALL

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0. Introduction

The purpose of the present paper is to give an elementary method for the computation of the cohomoloyg groups $H^q(X, \Omega^p(L))$, $(0 \leq q \leq n)$ of an n-dimensional Hopf manifold X, $(n \geq 2)$, where $\Omega^p(L)$ denotes the sheaf of germs of holomorphic p-forms with values in a holomorphic line bundle L on X.

Ise [10] has given a solution of this problem for homogeneous Hopf manifolds. He makes strong use of the fact that any homogeneous Hopf manifold of dimension n is a fibre bundle over the complex projective space P^{n-1}. His main tools are knowledge of the cohomology groups $H^q(P^{n-1}, \Omega^p(L))$ and the Leray spectral sequence.

Our approach to Ise's problem is a generalization of a method used by Douady [4] to study the deformation of Hopf manifolds.

We shall now describe the contents of this paper. Section 1 presents our method for the computation of the cohomology groups for flat line bundles and makes evident the dichotomy between Hopf manifolds of dimension $n>2$ and Hopf surfaces $(n=2)$. Section 2 treats the case of Hopf manifolds of dimension $n>2$ and section 3 the case of Hopf surfaces.

Generalizing a result of Kodaira [11] for Hopf surfaces we show in section 4 that all line bundles on an arbitrary Hopf manifold are flat. To this end we calculate first the Hodge numbers of these manifolds. The flatness of line bundles shows that the method introduced in section 1 applies to the general case.

Applications to holomorphic vector bundles and foliations on Hopf manifolds will be published elsewhere.

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1. Hopf manifolds, flat line bundles and the main lemma

Let $W := \boldsymbol{C}^n - \{0\}$ and $n \geq 2$. We recall some facts about Hopf manifolds,