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Osaka J. Math.
28 (1991), 985-997

REDUCIBILITY AND ORDERS OF PERIODIC AUTOMORPHISMS OF SURFACES

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(Received May 13, 1991)

Introduction

Det Σ_g be a closed oriented surface of genus $g\geq 2$. By an automorphism of Σ_g, we mean an element of the mapping class group $\mathcal{M}_g$ of Σ_g, which is the group of all isotopy classes of orientation preserving diffeomorphisms of Σ_g. The Nielsen-Thurston theory classifies the automorphisms of Σ_g into the following three types ([11]); (i) periodic, (ii) reducible, and (iii) pseudo-Anosov (the necessary definitions will be recalled in §1).

It is easy to see that the types (i) and (ii) have some overlap, although the type (iii) does not have any intersection with (i) nor (ii). The geometric characterization of this overlap was first obtained by Gilman [2] (Proposition 2.1). Recently, the author obtained the same characterization by a different approach making use of hyperbolic geometry ([4]).

In this paper, we apply the geometric characterization to consider the relationship between reducibility and orders of periodic automorphisms of Σ_g. Intuitively speaking, periodic automrophisms would tend to be irreducible when their orders grow since the number of components of an essential 1-submanifold, which should be invariant under reducible autuomorphisms, is known to be at most $3g-3$.

Recalling some definitions and necessary results, we shall proceed to justify the naive argument above by getting both the minimum order of periodic and irreducible automorphisms and the maximum order of periodic and reducible ones. The main result is given in §4. While the former value is obtained as a direct consequence of the geometric characterization, the latter requires some complicated calculations, all of which are elementary, however.

The author would like to express his sincere gratitude to Professor Shigeyuki Morita and Professor Sadayoshi Kojima for their suggestions and encouragements. He is also grateful to Professor Shigenori Matsumoto and Professor Yukio Matsumoto for their advices.

The author should acknowledge the referee unknown to him for pointing out that the geometric characterization is contained in a work of J. Gilman.