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μ-ELEMENTS IN S^1-TRANSFER IMAGES

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1. Introduction and Results

Let $\mu_r \in \pi^s_{8r+1}(S^0)$ be a μ-element of the $(8r+1)$-dimensional stable homotopy group of the sphere. Adams [1] gave a definition of μ_r, and showed that $d_R(\mu_r) \neq 0$ and

$$(1.1) \qquad \pi^s_{8r+1}(S^0) = Z/2\langle\mu_r\rangle \oplus \mathrm{Ker}(d_R) \qquad \text{for} \quad r \geq 0\,,$$

where $d_R\colon \pi^s_{8r+1}(S^0) \to \mathrm{Hom}(KO^0(S^0), KO^0(S^{8r+1}))$ is the d-invariant in the KO-theory. We assume that the mod 2 Adams filtration of μ_r is equal to $4r+1$, which determines each μ_r as a uniquely defined element.

Throughout the paper, CP^m_n denotes the suspension spectrum of a Thom complex $(CP^{m-n})^{n\xi}$ for $-\infty < n \leq m \leq \infty$ and $n \neq \infty$, where ξ is the canonical complex line bundle over the complex projective space CP^{m-n}. In [5] and [10], it is shown that, for $r>0$, μ_r is not in the image of the homomorphism t_*: $\pi^s_{8r}(CP^\infty_0) \to \pi^s_{8r+1}(S^0)$ induced from a stable map t called a S^1-transfer map. On the other hand, Knapp [9] investigated S^1-transfer maps $t_n\colon \Sigma^{-2n+1} CP^\infty_n \to S^0$, and proved that μ_r is in the image of $(t_2)_*$. We remark that $t_0 = t$. The purpose of the present paper is to discuss whether or not μ_r is in the image of $(t_n)_*$ for other values of n.

Let $I(\mu_r)$ be an ideal of $\pi^s_*(S^0)$ generated by μ_r. Then our main result is stated as follows:

Theorem 1. *Assume that $r \geq 0$. If $\binom{8r+2k+1}{4r+1} + 2\binom{8r+2k-1}{4r-1} \not\equiv 0 \bmod 4$ for an integer k, then*

$$I(\mu_r) \subset \mathrm{Im}\,[(t_{2k})_*\colon \pi^s_*(\Sigma^{-4k+1} CP^\infty_{2k}) \to \pi^s_*(S^0)]\,.$$

In contrast with Theorem 1, it holds that $\mu_r \notin \mathrm{Im}(t_{2k+1})_*$ for any $r \geq 0$ and k. More generally, if we treat μ_r with indeterminacy $\mathrm{Ker}(d_R)$, then it is possible to give a necessary and sufficient condition for our problem. In order to state it, we need some notations. Let a^j_i be the coefficient of x^i in the power series expansion of $((e^x-1)/x)^j$, and, for $n \leq m$,

$$(1.2) \qquad u(n, m) = \mathrm{Min}\,\{l>0 \mid l a^j_{m-j} \in Z \quad \text{for all } j \text{ with } n \leq j \leq m\}\,.$$