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THE EIGENVALUE DISTRIBUTION OF ELLIPTIC OPERATORS WITH HÖLDER CONTINUOUS COEFFICIENTS

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1. Introduction

In this paper we try to improve the estimates for the remainder term in the asymptotic formula for the eigenvalue distribution of an elliptic operator A of order $2m$ in $\boldsymbol{R}^n$ whose principal part has Hölder continuous coefficients with exponent τ, $0<\tau\leqq 1$. In some cases, mainly when $n=1$ we obtain a better estimate for the remainder term of the counting function $N(t)$ than the known results:

$$N(t)=\mu_A(\Omega)t^{n/2m}+O(t^{(n-\tau)/2m}) \qquad \text{as } t\to\infty.$$

In other cases it remains open whether our new estimate is valid or not. Instead of solving this problem we give a corresponding estimate for the remainder term in the asymptotic formula for the partition function (the trace of the heat kernel):

$$U(t)=\Gamma\Big(\frac{n}{2m}+1\Big)\mu_A(\Omega)t^{-n/2m}+O(t^{(\tau-n)/2m}) \qquad \text{as } t\to 0,$$

when $0<\tau<1$ and $2m>n$.

In order to describe the results more precisely and compare our results with the known ones we shall recall some standard notations and hypotheses. Let Ω be a domain in the n-dimensional Euclidean space $\boldsymbol{R}^n$ with a generic point $x=(x_1, \cdots, x_n)$. We denote by $\alpha=(\alpha_1, \cdots, \alpha_n)\in\boldsymbol{Z}_+^n$, $\boldsymbol{Z}_+=\{0, 1, 2, \cdots\}$ a multi-index of length $|\alpha|=\alpha_1+\cdots+\alpha_n$ and use the notations

$$\partial^\alpha=\partial_x^\alpha=\partial_1^{\alpha_1}\cdots\partial_n^{\alpha_n}, \quad \partial_k=\partial/\partial x_k,$$
$$D^\alpha=D_x^\alpha=D_1^{\alpha_1}\cdots D_n^{\alpha_n}, \quad D_k=-\sqrt{-1}\partial/\partial x_k.$$

For an integer $m\geqq 0$ $H^m(\Omega)$ is to be the set of all functions whose distributional derivatives of order up to m belong to $L_2(\Omega)$ and we introduce in it the usual norm