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PROPAGATION OF SINGULARITIES FOR HYPERBOLIC OPERATORS WITH MULTIPLE INVOLUTIVE CHARACTERISTICS

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0. Introduction

The aim of this paper is to study the propagation of C^∞-singularities for an hyperbolic pseudodifferential operator whose principal symbol vanishes at order $m\geq 2$ on an involutive manifold, generalizing a well known result obtained by R. Lascar [8] Chapter III, in the case $m=2$.

Let X be an open subset of $\boldsymbol{R}^{n+1}$, denote by $T^*X\cong X\times\boldsymbol{R}^{n+1}$ the cotangent bundle with canonical coordinates (x, ξ) and let $\omega=\sum_{j=0}^{n}\xi_j\,dx_j$ (resp. $\sigma=d\omega=\sum_{j=0}^{n}d\xi_j\wedge dx_j$) denote the canonical 1-form (resp. 2-form) on T^*X. By $T^*X\backslash 0$ we denote T^*X minus the zero section. Let $P(x, D_x)$ be a classical pseudo-differential operator (*pdo*) in X of order m, $m\in\boldsymbol{N}$, with symbol

$$p(x, \xi)\sim\sum_{j\geq 0}p_{m-j}(x, \xi)$$

and let $\varphi\in C^\infty(X)$ be a real-valued function, with $d\varphi(x)\neq 0\ \forall x\in X$.
We shall make the following assumptions:

(H_1) P is hyperbolic with respect to the level surfaces of φ, i.e. p_m is real-valued and
 i) $p_m(x, d\varphi(x))\neq 0\ \forall x\in X$;
 ii) for every $(x, \xi)\in T^*X$, ξ independent of $d\varphi(x)$, the function $p_m(x, \xi+td\varphi(x))$ is a polynomial of degree m in t having only real roots.

(H_2) There exists a C^∞-conic, non radial, involutive submanifold $N\subset T^*X\backslash 0$ of codimension $p+1$, such that, for $j\geq 0$, p_{m-j} vanishes at least of order $(m-2j)_+$ on $N(t_+=\max(t, 0))$.

The above conditions on N imply that, for any $\rho\in N$, we have $T_\rho(N)^\sigma\subset T_\rho(N)$ ($T_\rho(N)^\sigma$ being the orthogonal of $T_\rho(N)$ with respect to σ) and $\omega(\rho)\notin T_\rho(N)^\sigma$.
As a consequence, N is foliated by leaves F_ρ, $\rho\in N$, which are (immersed) C^∞ submanifold of N of dimension $p+1$ transversal to the radial vector field, with $T_\rho(F_\rho)=T_\rho(N)^\sigma$ (note that $p<n$). Moreover, for every $\rho\in N$, the bilinear form σ induces an isomorphism J_ρ: $T_\rho(T^*X)/T_\rho(N)\rightarrow T^*_\rho(F_\rho)$ (see [6]).