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A PROBABILISTIC CONSTRUCTION OF THE HEAT KERNEL FOR THE $\bar{\partial}$-NEUMANN PROBLEM ON A STRONGLY PSEUDOCONVEX SIEGEL DOMAIN

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1. Introduction

In this paper, we consider the heat equation for the $\bar{\partial}$-Neumann problem. This is an initial boundary value problem whose boundary condition includes *an imaginary directional differentiation.* In [7], Mallivain constructed the solution of the heat equation on a domain by using a method of singular perturbations and pointed out that a method related to the Fourier transform can be applied to this problem. On the other hand, a strongly pseudoconvex Siegel domain is well known as one of the most fundamental complex manifolds with boundary. This domain D can be regarded as the product of a Heisenberg group H_n and $\boldsymbol{R}^+$: $D=H_n\times\boldsymbol{R}^+$. In [4], Gaveau constructed explicitly the heat kernel for Kohn's Laplacian on H_n, by combining a probabilistic method and the Fourier transform. In this paper, by referring their works, we construct the heat kernel for the $\bar{\partial}$-Neumann problem on the Siegel domain D explicitly in terms of the theory of generalized Wiener functionals by Watanabe [13]. For the heat kernel on this domain, Stanton gave an explicit formula in the $(0, q)$-form case ($q>0$), by using methods of the partial differential equations [10], [11]. We here consider the general (p, q)-form case. The main part of our discussion is the proof of well-definedness of the heat kernel. In [12], our main results (Theorems 2.1 and 2.2 below) were announced.

We briefly explain our methods. The equation we consider is the following:

$$(1.1)\qquad \begin{cases} \dfrac{\partial}{\partial t}F(t,X)=-\Box F(t,X), \quad t>0,\ X\in D\,, \\ \lim\limits_{t\to 0}F(t,X)=f(X)\in\Lambda_0^{p,q}(D), \text{ uniformly on } \bar{D}\,, \\ PF(t,X)=Q\left(\dfrac{\partial}{\partial r}-i\dfrac{\partial}{\partial u}\right)F(t,X)=0 \text{ on } bD\,, \end{cases}$$

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