Fukushima, M. and Lejan, Y.
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ON QUASI-SUPPORTS OF SMOOTH MEASURES AND CLOSABILITY OF PRE-DIRICHLET FORMS

MASATOSHI FUKUSHIMA AND YVES LEJAN

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1. Introduction

Consider a singular perturbation of the Laplacian Δ on the d-dimensional Euclidean space R^d:

$$\hat{L} = -\Delta + L_\Gamma .$$

Here L_Γ is a linear operator "living on" a closed subset $\Gamma \subset R^d$ which might be of zero Lebesgue measure and as irregular as a fractal set. The problem is how and when we can give $\hat{L}$ a proper sense. One way to formulate this is to introduce a perturbed bilinear form

$$\hat{\mathcal{E}}(f, g) = \boldsymbol{D}(f, g) + \mathcal{E}_\Gamma(f|_\Gamma, g|_\Gamma), \qquad f, g \in C_0^\infty(R^d),$$

where $\boldsymbol{D}$ is the Dirichlet integral and $\mathcal{E}_\Gamma$ is a closable pre-Dirichlet form on $L^2(\Gamma; \mu)$ for some positive Radon measure μ on Γ such that $C_0^\infty(R^d)|_\Gamma \subset \mathcal{D}[\mathcal{E}_\Gamma]$. If $\hat{\mathcal{E}}$ is proven to be closable on $L^2(R^d)$, the L^2-space based on the Lebesgue measure dx, then the associated self-adjoint operator on $L^2(R^d)$ may be thought of as a realization of $\hat{L}$.

Some sufficient conditions for the closability of the perturbed pre-Dirichlet form $\hat{\mathcal{E}}$ are known (M. Fukushima [2; §2.1], J.F. Brasche and W. Karwowski [1]). It is plausible that $\hat{\mathcal{E}}$ ought to be closable on $L^2(R^d)$ under the sole potential theoretic assumption that μ charges no set of zero (Newtonian) capacity. A purpose of the present paper is to affirm this in a more general context as will be stated in §2 and proven in §4.

The proof in §4 involves the notion of the quasi-support of a measure and its characterization crucially. The quasi-notions have appeared in potential theory in diverse contexts. Another aim of the present paper is to show in §3 the existence of the quasi-support along with its useful characterizations in terms of classes of quasi-continuous functions.

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