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HEREDITARY RINGS AND RELATIVE PROJECTIVES

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We have given some characterizations of right Nakayama rings related to almost relative projectives or almost relative injectives [12]. In this paper we shall study particularly the condition (C) (resp (C^*)) in [12]. Let R be a right artinian ring and let M, N, U and V be R-modules. (C): M is almost N/N'-projective for any submodule N' of N, provided M is almost N-projective (resp. (C^*): U is almost V'-injective for any submodule V' of V, provided U is almost V-injective). We shall replace the role of N (resp. V) by that of M (resp. U) in the above.

We shall give several characterizations of semi-primary rings whose Jacobson radical is square-zero in the above manner and in the similar manner for relative projectives, respectively. Further from those viewpoints we shall characterize a certain type of hereditary rings over which every submodule of any indecomposable quasi-projective module is also quasi-projective (cf. [6]), and two-sided Nakayama rings with radical square-zero, respectively.

1. Relative projectives

In this paper we always assume that R is a ring with identity. Every module M is a unitary right R-module. We shall denote *the length, the Jacobson radical* and *an injective hull of* M by $|M|$, J(M) and E(M), respectively. By Soc(M) and $\mathrm{Soc}_i(M)$ we denote *the socle* and *the ith lower Loewy series of* M. We follow [4] and [11] for definitions of almost relative projectives and almost relative injectives.

In this section we study some conditions below, when M is N-projective for R-modules M and N (resp. U is V-injective for R-modules U and V).

(E) M/M' *is N-projective* and
(F) M' *is N-projective*

for any submodule M' of M, provided M is N-projective.
(resp.

(E^*) U' *is V-injective* and
(F^*) U/U' *is V-injective*