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DIRECT SUMS OF ALMOST RELATIVE INJECTIVE MODULES

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Let R be a ring with identity. When we study almost relative injective modules, the following problem is essential: Assume that an R-module V is almost U_j-injective for R-modules U_j ($j=1, 2, \cdots, n$), then *under what conditions is V also almost $\Sigma_j \oplus U_j$-injective?*

This problem is true without any assumptions, provided V is U_j-injective [2]. Y. Baba [3] gave an answer to the problem, when all V, U_j are uniform modules with finite length, and the author [6] generalized it to a case where the U_j are artinian indecomposable modules. Extending and utilizing the arguments given in [6], we shall drop the assumption "*artinian*" in this short note.

The proof will be completed by following the arguments given in [6]. Hence we shall explain only how we should modify the original proof in [6].

1. Preliminaries

Let R be a ring with identity. Every module in this paper is a right unitary R-module. We shall follow [3] and [6] for the terminologies. In [6], Theorem 2 we assumed that every module contained the non-zero socle. In this note we shall drop this assumption. Let W_1 and W_2 be R-modules. Take a diagram with V_2 a submodule of W_2:

$$\begin{array}{ccccc} W_2 & \overset{i}{\leftarrow} & V_2 & \leftarrow & 0 \\ & & \downarrow g & & \\ & & W_1 . & & \end{array} \tag{1}$$

Consider the following two conditions:

1) There exists $\tilde{g}\colon W_2 \to W_1$ such that $\tilde{g}\,|\,V_2=g$.

2) There exist a non-zero direct summand W of W_2: $W_2=W\oplus W'$ and $\tilde{g}\colon W_1 \to W$ such that $\tilde{g}g=\pi\,|\,V_2$, where π is the projection of W_2 onto W.

If either 1) or 2) holds true for any diagram (1), then we say that W_1 *is almost W_2-injective* (if 1) always holds true, then we say that W_1 *is W_2-injective* [2]).

We assume in the above that W_2 is indecomposable. If W_1 is almost