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A NOTE ON SURFACES WITH PENCILS OF NON-HYPERELLIPTIC CURVES OF GENUS 3.

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Introduction. Let $f: S \to B$ be a surjective holomorphic map between a nonsingular projective surface S and a nonsingular projective curve B of genus b. We always assume that it is relatively minimal, that is, there are no (-1)-curves in fibers of f. If a general fiber of f is a non-hyperelliptic curve of genus g, we call it a *non-hyperelliptic fibration of genus g*. The purpose of this paper is to state some results on surfaces with non-hyperelliptic fibration of genus 3.

In § 1, we shall give the lower bound on K^2 of such surfaces. More precisely, we have $K^2 \geq 3\chi(\mathcal{O}_S)+10(b-1)$. This was first obtained by Horikawa [7] and, later, by Chen [4]. Our proof is different from them and rather simple.

In § 2, we construct surfaces with non-hyperelliptic fibrations of genus 3. Though we restrict ourselves to regular surfaces here, our method can be applied to irregular ones as well (with some more effort). We remark that the other examples of such srufaces can be found in [1].

To explain the background of the construction, let $f: S \to B$ be a non-hyperelliptic fibration of genus 3. Then, we have a canonical birational map of S into a $\boldsymbol{P}^2$-bundle W on B (see, § 1 below). We let V be its image, and consider the fibration $f': V \to B$ induced by the projection map of W. The difference of the invariants $(\chi(\mathcal{O}_V)-\chi(\mathcal{O}_S),\ \omega_V^2-\omega_S^2)$ can be considered as the contribution of singular fibers of f'. Though we do not have a complete list of possible singular fibers, we at least can expect that they are quite similar to those in [8]. However, a singular fiber arising from a simple elliptic singularity of type $\tilde{E}_7$, which Ashikaga has constructed in [1], seems to be a "special" one ([8, § 9]). What this means may be guaranteed by the fact that the canonical bundle of S cannot be ample in this case. Therefore, there sholud be a way to construct "general" ones. This is the motivation of the present construction.

In § 3, we shall give examples of Type I degenerations, extending a result in [3] and [5]. We hope that such mild degenerations can be used to attack the Torelli problem via degenerate loci (see, [10] for such an approach).

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