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THE CHERN CHARACTER HOMOMORPHISM OF THE COMPACT SIMPLY CONNECTED EXCEPTIONAL GROUP E_6

Dedicated to Professor Shôrô Araki on his sixtieth birthday

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0. Introduction

Let F_4 and E_6 be the compact, 1-connected representatives of the respective local classes. As in [22] there is an involutive automorphism θ of E_6 such that the subgroup consisting of fixed points of θ is F_4. Thus the quotient E_6/F_4 forms a compact symmetric space, which is denoted by EIV in É. Cartan's notation. For brevity we shall write EIV instead of E_6/F_4.

The ordinary cohomology and complex K-theory of three spaces F_4, E_6 and EIV are well understood (see §1). Moreover, the Chern character homomorphism of F_4 was described explicitly in [20]. The purpose of this paper is to study those of E_6 and EIV. Our results are stated as follows (for notations used below, see §1):

Theorem 1. *The Chern character homomorphism*

$$ch\colon K^*(E_6) = \Lambda_Z(\beta(\rho_1), \beta(\rho_2), \beta(\Lambda^2\rho_1), \beta(\Lambda^3\rho_1), \beta(\Lambda^2\rho_6), \beta(\rho_6)) \to H^*(E_6; Q) = \Lambda_Q(x_3, x_9, x_{11}, x_{15}, x_{17}, x_{23})$$

is given by

$$ch(\beta(\rho_1)) = 6x_3+\frac{1}{2}x_9+\frac{1}{20}x_{11}+\frac{1}{168}x_{15}+\frac{1}{480}x_{17}+\frac{1}{443520}x_{23}$$

$$ch(\beta(\rho_2)) = 24x_3-\frac{3}{10}x_{11}+\frac{3}{28}x_{15}-\frac{31}{221760}x_{23}$$

$$ch(\beta(\Lambda^2\rho_1)) = 150\,x_3+\frac{11}{2}x_9-\frac{1}{4}x_{11}-\frac{101}{168}x_{15}-\frac{229}{480}x_{17}-\frac{2021}{443520}x_{23}$$

$$ch(\beta(\Lambda^3\rho_1)) = 1800\,x_3-\frac{27}{2}x_{11}-\frac{153}{28}x_{15}+\frac{6789}{24640}x_{23}$$

$$ch(\beta(\Lambda^2\rho_6)) = 150\,x_3-\frac{11}{2}x_9-\frac{1}{4}x_{11}-\frac{101}{168}x_{15}+\frac{229}{480}x_{17}-\frac{2021}{443520}x_{23}$$

$$ch(\beta(\rho_6)) = 6\,x_3-\frac{1}{2}x_9+\frac{1}{20}x_{11}+\frac{1}{168}x_{15}-\frac{1}{480}x_{17}+\frac{1}{443520}x_{23}\,.$$