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ON IMBEDDING 3-MANIFOLDS INTO 4-MANIFOLDS

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Introduction

We discuss an imbedding problem of a closed, connected, oriented 3-manifold into a given compact connected 4-manifold, which arises from certain signature invariants of 3-manifold associated with its cyclic coverings. Our main result is the following:

Theorem. *For any compact, connected (orientable or non-orientable) 4-manifold W (with or without boundary), there exist infinitely many closed, connected, orientable 3-manifolds M which cannot be imbedded in W.*

For a closed orientable 4-manifold W, this is a direct consequence of [**8**, Theorem 3.2] and, for an orientable 4-manifold W with boundary, we can prove it by using the doubling technique for W. Thus the main concern in this paper is for a non-orientable 4-manifold W.

The proof of Theorem is given in §3. In §2, a classification of the types of imbeddings of M into a closed 4-manifold W is given. Section 1 is devoted to the calculation of the signatures of the finite cyclic covers of a homology handle M. We can express these signatures in terms of the local signatures of M under a certain condition on the Alexander polynomial of M, where the Alexander polynomial of a homology handle is defined in the same way as in the case of knots (*cf.* [**3**, Definition 1.3]). Let $\sigma_a(M)$ be the local signature of M at $a\in[-1, 1]$, which is an analogue of the Milnor signature of a knot (*cf.* [**9**]). Let $\sigma^{(n)}(M)$ be the signature of n-fold cyclic cover of M (whose definition is given in Section 1 where $\sigma^{(n)}(M)$ is denoted by $\sigma^{\dot{\gamma}^{(n)}}(M_{\dot{\gamma}(n)})$). Then the following will be shown.

Proposition 1.3. *If the Alexander polynomial of M has no $2n$-th root of unity, then*

$$\sigma^{(n)}(M) = \sum_{j=0}^{n-1} (-1)^j \sum_{a_{j+1}<a<a_j} \sigma_a(M)\,,$$