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THREE-FOLD IRREGULAR BRANCHED COVERINGS OF SOME SPATIAL GRAPHS

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1. Introduction

A *spatial graph* is a graph embedded in a 3-sphere S^3. In this paper, we consider three-fold irregular branched coverings of some spatial graphs. In particular, we investigate those of some of θ-curves and handcuff graphs in S^3 and prove that there exists at least one three-fold irregular branched covering of these graphs. Further, we identify these branched coverings. Hilden [4] and Montesinos [6] independently showed that every orientable closed 3-manifold is a three-fold irregular covering of S^3, branched along a link.

Let L be a spatial graph and $G=\pi_1(S^3-L)$. Then there is a one-to-one correspondence between n-fold unbranched coverings of S^3-L and conjugacy classes of transitive representations of G into S_n, the symmetric group with n letters $\{0, 1, \cdots, n-1\}$. Let μ be such a representation, called a *monodromy map*, and $T=\mu(G)$. Define T_0 as the subgroup of T that fixes letter 0. Then $\mu^{-1}(T_0)$ is the fundamental group of the unbranched covering associated with μ. To each unbranched covering of S^3-L there exists the unique completion $\tilde{M}_\mu(L)$ called the associated branched covering (see Fox [1]).

In this paper we investigate a monodromy map $\mu: G\to S_3$ which is surjective, i.e. the covering is irregular. We call μ an *S_3-representation* of L. Further we only consider the case that the branched covering associated with μ is an orientable 3-manifold.

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2. Three-fold branched coverings of spatial θ-curves

In this section, let L denote a spatial θ-curve that consists of three egdes e_1, e_2 and e_3, each of which has distinct endpoints A and B. Suppose that each of e_1, e_2 and e_3 is oriented from A to B. Then $G=\pi_1(S^3-L)$ is generated by $x_1, \cdots, x_l$; $y_1, \cdots, y_m$; $z_1, \cdots, z_n$, where each of x_i, y_j and z_k corresponds to a meridian of each of e_1, e_2 and e_3, respectively. Note that every element of S_3 can be expressed as $a^\delta b^\varepsilon$, where $a=(01)$, $b=(012)$; $\delta=0, 1, \varepsilon=0, 1, 2$. We assume that