Shimizu, S.
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HOLOMORPHIC EQUIVALENCE PROBLEM FOR A CERTAIN CLASS OF UNBOUNDED REINHARDT DOMAINS IN C^2

SATORU SHIMIZU

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Introduction

An answer to the holomorphic equivalence problem for arbitrary bounded Reinhardt domains was given in [1]. It seems interesting to investigate whether a similar result on unbounded Reinhardt domains holds or not.

Now, for a pair (a, b) of non-negative real constants with $(a, b) \neq (0, 0)$ and a positive real constant r, let us consider an unbounded Reinhardt domain $D_{a,b}(r)$ in C^2 given by

$$D_{a,b}(r) = \{(z, w) \in C^2 \mid |z|^a |w|^b < r\}.$$

Here, when $ab=0$, for example, when $b=0$, the domain $D_{a,0}(r)$ is understood as

$$D_{a,0}(r) = \{(z, w) \in C^2 \mid |z|^a < r\}.$$

We are concerned with the holomorphic automorphisms and the equivalence of the domains $D_{a,b}(r)$. In the present paper, we confine ourselves to the case where a and b are integers. The case where a and b are arbitary non-negative real constants will be treated in the subsequent paper [3].

Our main result of this paper can be stated as the following theorem.

Theorem. *If a domain $D_{a,b}(r)$ with $(a, b) \in (\mathbf{Z}_{\geq 0})^2$ is biholomorphic to a domain $D_{u,v}(s)$ with $(u, v) \in (\mathbf{Z}_{\geq 0})^2$, then there exists a transformation φ given by*

$$\varphi\colon C^2 \ni (z, w) \mapsto (\alpha z, \beta w) \in C^2$$

or

$$\varphi\colon C^2 \ni (z, w) \mapsto (\gamma w, \delta z) \in C^2$$

such that $\varphi(D_{a,b}(r)) = D_{u,v}(s)$, where $\mathbf{Z}_{\geq 0}$ denotes the set of non-negative intergers and $\alpha, \beta, \gamma, \delta$ are non-zero complex constants.

This paper is organized as follows. In Section 1, we recall basic concepts and results on Reinhardt domains. In particular, we give a general formulation of the holomorphic equivalence problem for Reinhardt domains as well as the