

NONSTANDARD ANALYSIS OF LINEAR CANONICAL TRANSFORMATIONS ON A FERMION FOCK SPACE WITH AN INDEFINITE METRIC

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0. Introduction

It is well known that an indefinite metric Hilbert space is necessary in order to describe the quantum electromagnetic field (See Strocchi, F. and A.S. Wightman [11]). Ito, K.R. [5] investigated two dimensional quantum electrodynamics in the indefinite metric formulation, where the theory of linear canonical transformation on Boson Fock space with an indefinite metric was used which was developed in Ito, K.R. [4]. On the other hand, indefinite metric Hilbert space are not necessary for Dirac field in usual formulation. But in the Euclidean formulation sometimes appears an indefinite metric Hilbert space. Nagamachi, S. and N. Mugibayashi [7] studied the Euclidean formulation of Dirac field and its Euclidean covariance. There appeared a Fermion Fock space with an indefinite metric and canonical transformations which represent Euclidean transformations of field operators. Fortunately, since these canonical transformations do not mix creation and annihilation operators and moreover the operators Φ , Ψ which determine the canonical transformation commute with the operator η giving the indefinite metric in the form $[x, y] = (x, \eta y)$, they are implementable by bounded operators which are isometric with respect to the indefinite metric, which we call Λ -unitary operators (Remark 7.10). In generalizing the theory of Clifford group of Sato, M., T. Miwa and M. Jimbo [17] to an infinite dimensional case, Palmer, J. [8] found the condition under which an automorphism of Clifford algebra is implementable by some operator in the Fock space. Similar results were obtained by Araki, H. [1]. Their results have an intimate connection with ours but do not concern the implementability by an isometry operator with respect to the indefinite metric inner product which we call a Λ -unitary operator.

In this paper, we extensively use nonstandard analysis and Berezin calculus to investigate the linear canonical transformations in an infinite dimensional Fermion Fock space with an indefinite metric, especially their implementability by a Λ -unitary operator. In the same time we want to show how the Berezin