Morioka, T.
Osaka J. Math.
28 (1991), 563–578

HYPOELLIPTICITY FOR SEMI-ELLIPTIC OPERATORS WHICH DEGENERATE ON HYPERSURFACE

TATSUSHI MORIOKA

(Received August 9, 1990)

1. Introduction and results

Let us denote a coordinate of $T^*(\boldsymbol{R}^n)$ by the following notation:

$$T^*(\boldsymbol{R}^n) = \{(x, y; \xi, \eta): x, \xi \in \boldsymbol{R}^{n_1} \quad \text{and} \quad y, \eta \in \boldsymbol{R}^{n_2}\} .$$

Here $n=n_1+n_2$. In this paper, we shall study the hypoellipticity of semi-elliptic operators in $\boldsymbol{R}^n$ which degenerate at $x=0$. It is well known that non-degenerate semi-elliptic operators are hypoelliptic. For the definition of semi-elliptic operators, see Kumano-go [5, p.85]. We consider a differential operator of the form

$$L = a(x, y, D_x)+g(x)\, b(x, y, D_y) \quad \text{in } \boldsymbol{R}^n = \boldsymbol{R}^{n_1}_x \times \boldsymbol{R}^{n_2}_y , \tag{1.1}$$

satisfying the following conditions. (Throughout this paper, the coefficients of differential operators are assumed to be functions of the class C^∞.)

(A.1) $g(0)=0$ and $g(x)>0$ for $x \neq 0$.
(A.2) $a(x, y, D_x)$ is a differential operator of order $2l$ and

$$\mathrm{Re}\, a(x, y, \xi) \geqq C_1 |\xi|^{2l}$$

holds for sufficiently large $|\xi|$.
(A.3) $b(x, y, D_y)$ is a differential operator of order $2m$ and

$$\mathrm{Re}\, b(x, y, \eta) \geqq C_2 |\eta|^{2m}$$

holds for sufficiently large $|\eta|$. Here C_1 and C_2 are positive constants and l, m are positive integers.

Our main result is the following:

Theorem 1. *Let L be an operator of the form* (1.1) *satisfying* (*A*.1)–(*A*.3). *Then L is hypoelliptic, i.e.,*

$$\text{sing supp } Lu = \text{sing supp } u \quad \text{for} \quad u \in \mathcal{D}' .$$

Taniguchi [12] showed that L is hypoelliptic if $g(x)$ is non-negative and