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ON THE CLOSABLE PARTS OF PRE-DIRICHLET FORMS AND THE FINE SUPPORTS OF UNDERLYING MEASURES

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1. Introduction

Let X be a locally compact separable metric space, $\mathcal{M}$ be the space of positive Radon measures on X and let $\mathcal{M}'=\{\nu\in\mathcal{M}: supp[\nu]=X\}$. Fix $m\in\mathcal{M}'$ and a regular Dirichlet form $\mathcal{E}$ with domain $\mathcal{F}$ on $L^2(X; m)$, which possesses a nice core $\mathcal{C}$ as described in Section 3. Throughout the present paper, we assume that $\mathcal{E}$ is either irreducible or transient. Let $Cap(\cdot)$ be the 1-capacity associated with $\mathcal{E}$. A set A is said to be $\mathcal{E}_1$-polar if $Cap(A)=0$. Define

$$\mathcal{M}_0 = \{\nu\in\mathcal{M}\colon \nu \text{ charges no } \mathcal{E}_1\text{-polar set}\},$$
$$\mathcal{M}_{00} = \{\nu\in\mathcal{M}_0\colon Cap(X\backslash\tilde{S}_\nu) = 0\},$$

where $\tilde{S}_\nu$ stands for the support of the positive continuous additive functional (abbreviated to PCAF) associated with $\nu\in\mathcal{M}_0$. $\tilde{S}_\nu$ is closed with respect to the fine topology for the associated Hunt process and we call it the fine support of ν.

For $\mu\in\mathcal{M}$, we introduce the capacitary decomposition of μ_1 with respect to $Cap(\cdot)$: a unique decomposition $\mu=\mu_0+\mu_1$, where $\mu_0\in\mathcal{M}_0$ and $\mu_1=\boldsymbol{I}_N\cdot\mu$ with an $\mathcal{E}_1$-polar set N. For details, see Section 2. This is a variant of the potential-theoretical decomposition of measures due to Blumenthal-Getoor [1, VI(3.6)].

In the present paper, we are interested in changing the underlying measure m for another element of $\mathcal{M}'$ by keeping the pre-Dirichlet form $\mathcal{E}$ on $\mathcal{C}$ unchanged. We aim at showing the following necessary and sufficient condition for $\mu\in\mathcal{M}'$,

$$(\mathcal{E}, \mathcal{C}) \textit{ is closable on } L^2(X; \mu) \textit{ if and only if } \mu_0\in\mathcal{M}_{00}\,.$$

See Theorem 4.1, where the Hunt process associated with the closure is also specified by time changing with respect to μ_0 and making points of N traps.

The condition that $\nu\in\mathcal{M}_{00}$ is an indispensable requirement for the invariance of the pre-Dirichlet form under the random time change with respect to