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REGULARITY PROPERTIES FOR STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS OF PARABOLIC TYPE

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1. Introduction

We consider the following semilinear stochastic partial differential equation (SPDE) of parabolic type:

$$(1.1)\qquad dS_t(x) = -\mathcal{A}S_t(x)dt+\sum_{j=1}^{J}\mathcal{B}_j\{B_j(x, S_t)\}dt+\sum_{j=1}^{J}C_j dw_t^j(x)\,, \qquad x\in \boldsymbol{R}^d,\quad t>0\,.$$

Here $\mathcal{A}=\sum_{|\alpha|\leq 2m}a_\alpha D^\alpha$, $\mathcal{B}_j=\sum_{|\alpha|\leq n}b_{j,\alpha}D^\alpha$ and $C_j=\sum_{|\alpha|\leq l}c_{j,\alpha}D^\alpha$, $m\geq 1, n, l\geq 0$, are differential operators with coefficients $a_\alpha, b_{j,\alpha}, c_{j,\alpha}\in C_b^\infty(\boldsymbol{R}^d)$, $1\leq j\leq J$, and $\{B_j(x, S)\}_{j=1}^J$ are certain functions of x and $S=\{S(x); x\in\boldsymbol{R}^d\}$. We denote $D^\alpha \equiv D_x^\alpha=\left(\frac{\partial}{\partial x_1}\right)^{\alpha_1}\cdots\left(\frac{\partial}{\partial x_d}\right)^{\alpha_d}$ and $|\alpha|=\sum_{i=1}^d\alpha_i$ for $\alpha=(\alpha_1\cdots, \alpha_d)\in\boldsymbol{Z}_+^d=\{0, 1, 2, \cdots\}^d$, while $C_b^\infty(\boldsymbol{R}^d)$ stands for the class of all C^∞-functions on $\boldsymbol{R}^d$ possessing bounded derivatives of all orders. The system $\{w_t^j(x)\}_{j=1}^J$ consists of J independent $\{\mathcal{F}_t\}$-cylindrical Brownian motions (c.B.m.'s) ([6], [7]) on the space $L^2(\boldsymbol{R}^d)$ which are defined on an appropriate probability space $(\Omega, \mathcal{F}, P)$ equipped with a reference family $\{\mathcal{F}_t\}$.

The general theory for the SPDE's has been developed by several authors based mainly on two different approaches, namely, the semigroup method (e.g. Dawson [4]) and the variational one (e.g. Pardoux [14], Krylov and Rozovskii [12]). It is actually possible to establish the existence and uniqueness of solutions to (1.1) by employing these former results; see Remark 2.2 below. However, in order to continue further investigation of the behavior of solutions, the meaning of solutions due to their theory happens not to be sufficiently strong. In other words, as a rule, they sometimes require too large space for solutions. The main purpose of this article is to fill this gap up by showing that the solutions live on *nice* spaces. This will be accomplished by studying the regularity properties, strong and weak differentiability, of solutions of (1.1).

Let us now introduce the state spaces for the solutions S_t of (1.1). A positive function $\chi\in C^\infty(\boldsymbol{R}^d)$ satisfying $\chi(x)=|x|$ for x; $|x|\geq 1$ and $\chi(-x)=\chi(x)$