Funaki, T.
Osaka J. Math.
28 (1991) 463–494

THE REVERSIBLE MEASURES OF MULTI-DIMENSIONAL GINZBURG-LANDAU TYPE CONTINUUM MODEL

TADAHISA FUNAKI

(Received June 2, 1990)

1. Introduction

In this paper we investigate a stochastic dynamics for continuum fields on $\boldsymbol{R}^d$ with interactions prescribed by Ginzburg-Landau type Hamiltonian. The main problems discussed here are to clarify the structure of the family of reversible measures (r.m.'s) of this dynamics, especially, we are interested in (1) the characterization, (2) the construction and (3) showing the uniqueness of r.m.'s. For the characterization problem the classical notion of Gibbs states (e.g., for the lattice systems) is extended to the continuum fields. In our present situation Gibbs states are Markovian random fields over $\boldsymbol{R}^d$ and they are given as local perturbations from Gaussian fields, which is determined by the so-called DLR equation. Then the answer to the first problem will be given by establishing the equivalence between reversibility and Gibbs property. The r.m.'s and therefore the Gibbs states will be constructed for a wide class of potentials, while for the uniqueness we require the strict-convexity for the self-potential appearing in the Hamiltonian. In this uniqueness domain, we also verify the strong mixing property of the Gibbs states. This is one of examples which show stochastic dynamics is useful for the study of properties of Gibbs states.

Now let us explain the dynamics we shall discuss in this paper more explicitly. It is described by the so-called time-dependent Ginzburg-Landau equation (TDGL eq.) of non-conservative type:

$$(1.1) \qquad dS_t(x) = -\frac{1}{2} D\mathcal{H}(x, S_t)\, dt + dw_t(x), \quad t>0,\ x\in \boldsymbol{R}^d,$$

where w_t is a cylindrical Brownian motion (c.B.m.) on $L^2(\boldsymbol{R}^d)$, see [7, 8]. The solution S_t determines a random time evolution of real-valued continuum field on $\boldsymbol{R}^d$. The Hamiltonian $\mathcal{H}$ is formally given as the sum of two terms, local-interaction and self-interaction:

$$(1.2) \qquad \mathcal{H}(S) = \int_{\boldsymbol{R}^d} \{\frac{1}{2} \mathcal{A}S(x)\cdot S(x) + V(x, S(x))\}\, dx, \quad S\colon \boldsymbol{R}^d \to \boldsymbol{R}.$$