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SUPER G-STRUCTURES OF FINITE TYPE

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0. Introduction

Many important differential geometric structures on manifolds such as Riemannian metrics, complex structures etc, can be grasped as G-structures. A G-structure is defined as a reduction of the structure group of the linear frame bundle of a manifold to a linear Lie group G. In other words, a G-structure on a manifold can be regarded as a system of partial differential equations of first order for local coordinates of the manifold. This formulation of G-structures can be generalized to supermanifolds.

Some of the concrete differential geometric structures which can be regarded as G-structures defined on supermanifolds, have been studied extensively. For example, many authors studied Reimannian supermanifolds (cf. [11]), since metric is one of the most fundamental objects in geometry and physics. Super Hamiltonian structure was formulated by Kostant ([5]) in order to study the geometric quantization on supermanifolds. Super CR-structure (a structure on a real subsupermanifold induced from an ambient complex superspace) was studied by Schwarz ([10]) and by Rosly and Schwarz ([6] and [7]) in relevance to supergravity.

The notion of G-structure on supermanifolds was first introduced in [10] and used in [6] and [7] although general theory of G-structures on supermanifolds is not developed there.

In this paper, we investigate general G-structures of finite type on supermaniolds and show the following theorem:

Main Theorem. *Let G be a linear Lie supergroup of finite type with the connected body. Then the equivalence problem of G-structures can be reduced to the equivalence problem of complete parallelisms, that is, $\{e\}$-structures.*

Our main theorem is a generalization of a well-known theorem (Theorem 0 in § 1) concerning manifolds and the outline of the proof goes similarly as that of usual manifolds. Many facts in super-geometry are formally the same as in the usual geometry, although their verifications are mostly non-trivial.