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PROPER DUPIN HYPERSURFACES GENERATED BY SYMMETRIC SUBMANIFOLDS

Dedicated to Professor Tadashi Nagano on his sixtieth birthday

MASARU TAKEUCHI

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Introduction

A connected oriented hypersurface M of the space form $\bar{M}=E^n$, S^n or H^n is called a *Dupin hypersurface*, if for any curvature submanifold S of M the corresponding principal curvature λ is constant along S. Here by a *curvature submanifold* we mean a connected submanifold S with a smooth function λ on S such that for each point $x\in S$, $\lambda(x)$ is a principal curvature of M at x and T_xS is equal to the principal subspace in T_xM corresponding to $\lambda(x)$. A Dupin hypersurface is said to be *proper*, if all principal curvatures have locally constant multiplicities. A connected oriented hypersurface of $\bar{M}$ is called an *isoparametric hypersurface*, if all principal curvatures are locally constant. Obviously an isoparametric hypersurface is a proper Dupin hypersurface. Another example of a Dupin hypersurface (Pinkall [6]) is an ε-tube M^ε around a symmetric submanifold M of $\bar{M}$ of codimension greater than 1, which is said to *be generated by M*. Recall that a connected submanifold M of $\bar{M}$ is a *symmetric submanifold*, if for each point $x\in M$ there is an involutive isometry σ of $\bar{M}$ levaing M and x invariant such that (-1)-eigenspace of $(\sigma_*)_x$ is equal to T_xM. The most simple example is the tube M^ε around a complete totally geodesic submanifold M. This is a complete isoparametric hypersurface with two principal curvatures, which is further *homogeneous* in the sense that the group

$$\mathrm{Aut}(M^\varepsilon)=\{\phi\in I(\bar{M});\ \phi(M^\varepsilon)=M^\varepsilon\}$$

acts transitively on M^ε. Here $I(\bar{M})$ denotes the group of isometries of $\bar{M}$. In this note we will determine all the symmetric submanifolds whose tube is a proper Dupin hypersurface, in the following theorem.

Theorem. *Let M be a non-totally geodesic symmetric submanifold of a space form $\bar{M}$ of codimension greater than* 1. *Then the tube M^ε around M is a proper Dupin hypersurface if and only if either*
(i) *M is a complete extrinsic sphere of $\bar{M}$* (*see Section* 2 *for definition*) *of codimen-*