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ON THE EXISTENCE OF UNRAMIFIED p-EXTENSIONS

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Introduction

Let p be an odd prime. Let K be an algebraic number field of finite degree, and let L/K be a p-extension. Throughout this paper, a p-extension means a finite Galois extension whose Galois group is a p-group. In this paper, we study the existence of a p-extension $M/L/K$ such that M/L is unramified.

One of our results is the following.

Let k be the rational number field or an imaginary quadratic field with the class number prime to p (p is not equal to 3 when $k=\boldsymbol{Q}(\sqrt{-3})$). Let $L/K/k$ be a Galois tower satisfying the conditions (1), (2) and (3) in Theorem 1 below, and E be a non-split central extension of $\mathrm{Gal}(L/k)$ by $\boldsymbol{Z}/p\boldsymbol{Z}$. Then there exists a Galois extension M/k such that M/K is unramified and $\mathrm{Gal}(M/k)$ is isomorphic to E.

We try to proceed by means of the theory of central imbedding problems. In §**1**, we explain about the central imbedding problems. In §**2**, we study the existence of unramified p-extensions, and in §**3** and §**4**, we have an application of results proved in §**2**. In §**5**, we study the central imbedding problem of exponent p.

1. Central imbedding problems

Let k be an algebraic number field of finite degree, $\mathfrak{G}$ its absolute Galois group, and let L/k be a finite Galois extension with Galois group G. Let (ε): $1\to A\to E\xrightarrow{j} G\to 1$ be a central extension of finite groups. Then a central imbedding problem $(L/k, \varepsilon)$ is defined by the diagram

(*)
$$\begin{array}{ccccccccc} & & & & & & \mathfrak{G} & & \\ & & & & & & \downarrow{\scriptstyle\varphi} & & \\ (\varepsilon)\colon 1 & \longrightarrow & A & \longrightarrow & E & \xrightarrow{j} & G & \longrightarrow & 1 \end{array}$$

where φ is the canonical surjection. A solution of the imbedding problem $(L/k,\varepsilon)$ is, by definition, a continuous homomorphism ψ of $\mathfrak{G}$ to E with $j\circ\psi=\varphi$. The