Kawai, H.
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ON THE COHOMOLOGY OF FINITE GROUPS AND THE APPLICATIONS TO MODULAR REPRESENTATIONS

HIROAKI KAWAI

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1. Introduction

Let G be a finite group and k be a field of prime characteristic p. All modules considered here are assumed to be finite dimensional over k. In [2], Carslon introduced a certain condition on the cohomology ring of G to study the structure of periodic modules by homological techniques. Let us denote it by $C(n)$, where n is a positive integer. If G satisfies $C(n)$, then there are homogeneous elements of degree n having an interesting property releted to his notion of rank variety (see Section 3 for details). For a p-group P, he showed that there exists an integer $n(P)$ such that P satisfies $C(2n(P))$. And using this, he showed that the period of a periodic kP-module divides $2n(P)$.

The purpose of this paper is to extend Carlson's results to an arbitrary finite group G. In doing so, we shall give a stronger version of the condition $C(n)$, with a couple of equivalent conditions to it. Concerning Carlson's number $n(G)$ which can as well be defined for an arbitrary G, we shall prove that there exist cohomology elements of degree $2n(G)$ satisfying our new condition, so that G satisfies $C(2n(G))$. As an application of this result, we shall show that the period of a periodic kG-module divides $2n(G)$. As another application, we also give a homoligical criterion for a kG-module to be projective. A similar criterion has been given by Donovan [6], in response to a problem of Schultz [9].

2. Preliminaries

In this section we mention some preliminary facts needed in later arguments. For a kG-module M, set $\mathrm{Ext}^*_{kG}(M, M)=\sum_{n\geq 0}\mathrm{Ext}^n_{kG}(M, M)$. If H is a subgroup of G, then M_H denotes the restriction of M to a kH-module. First of all, we prove the following general fact.

Proposition 2.1. *Let* $0\rightarrow N_1\rightarrow M\xrightarrow{f}L\rightarrow 0$ *and* $0\rightarrow N_2\rightarrow M\xrightarrow{g}L\rightarrow 0$ *be exact*