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GROWTH OF EQUIVARIANT HARMONIC MAPS AND HARMONIC MORPHISMS

Dedicated to Prof. Tadashi Nagano on his 60th birthday

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0. Introduction

The purpose of the present paper is to study the growth of certain harmonic maps in relation with the geometry of the domains and ranges.

Let $\phi: M \to N$ be a harmonic map between complete noncompact Riemannian manifolds M and N. We fix a point o of M (resp. a point o' of N) and denote by r_M (resp. r_N) the distance to o in N (resp. o' in N). Set $\mu(\phi; t) := \max\{r_N(\phi(x)) : x \in M, r_M(x) = t\}$. We want to know the growth of ϕ, or the asymptotic behavior of $\mu(\phi; t)$ as t goes to infinity. We first recall the following result by Cheng [8] (cf. also [3] [31: Chap. 6]): Suppose that M has nonnegative Ricci curvature and N is a Hadamard manifold, namely, N is a simply connected and nonpositively curved manofod manifold. Then the energy density $e(\phi)$ of the map ϕ satisfies: $e(\phi)(o) \leq c_m \mu(\phi: t)^2 t^2$, where c_m is a constant depending only on the dimension m of M. It follows that ϕ is a constant map if ϕ has sublinear growth, that is, $\liminf_{t\to\infty} \mu(\phi; t)/t = 0$. We are interested in a (nonconstant) harmonic map $\phi: M \to N$ which has linear growth, namely, which has the property that $\limsup_{t\to\infty} \mu(\phi; t)/t < +\infty$. For instance, it turns out that a harmonic map $\phi: M \to N$ of linear growth must be totally geodesic if M has volume growth of at most quadratic order (cf. [9]). It has been also proved in [24] that a d-closed harmonic 1-form of bounded length on M must be parallel if the sectional curvature of M is nonnegative and decays quadratically. Moreover Li and Tam [26] have shown that the dimension on the space of linear growth harmonic functions on M is less than or equal to $k+1$ if the volume of the metric ball of radius t around o is bounded by ct^k for some constant c.

On the other hand, we can construct a noncompact complete manifold M of positive Ricci curvature and a harmonic map $\phi_F: M \to F$ of bounded energy density from M onto a complete manifold F of nonnegative Ricci curvature (cf. Example in Section 2). It turns out from the construction that ϕ is a *harmonic marphism* from M onto F with *totally geodesic fibers*, namely, it is a