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STABILITY OF SURFACES WITH CONSTANT MEAN CURVATURE IN 3-DIMENSIONAL RIEMANNIAN MANIFOLDS

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0. Introduction

In [1] Barbosa and do Carmo adopted a new approach to the stability of minimal surfaces. In particular they discussed the stability of simply connected compact domains with boundary on minimal surfaces in space forms (cf. [2], [8] and [10]). Their method was applied also to the stability of surfaces with constant mean curvature in 3-dimensional space forms (see [5], [9] and [14]). It is natural to ask if these arguments can be generalized for general ambient spaces. In the case of minimal surfaces, a positive answer to this question was given in our previous paper [11]. In this paper we give a positive answer in the case of surfaces with constnat mean curvature. Namely we prove:

Theorem. *Let $f: M \rightarrow N$ be an immersion of a 2-dimensional orientable manifold M into a 3-dimensional orientable Riemannian manifold N. Assume that the mean curvature of the immersion f is constant. Let D be a simply connected compact domain on M with piecewise smooth boundary. We denote by A and dM the second fundamental form and the area element of M induced by f, respectively. Suppose that the sectional curvature of N and the norm of the covariant derivative of the curvature tensor of N are bounded. Then there is a positive constant c_1 depending only on N such that if $\int_D (1+|A|^2/2)dM<c_1$, then D is stable.*

REMARK. (i) The method in [11] is not available to the statility of surfaces with constant mean curvature.

(ii) If we omit the hypothesis that D is simply connected. it is not known whether the theorem is ture or not (cf. [11, Theorem 0.3]).

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1. Preliminaries

Let $f: M \rightarrow N$ be an immersion of an m-dimensional orientable manifold