Cheng, Q.M.
Osaka J. Math.
27 (1990), 885-892

COMPLETE MINIMAL HYPERSURFACES IN $S^4(1)$ WITH CONSTANT SCALAR CURVATURE

QING-MING CHENG

(Received October 31, 1989)

1. Introduction

Let M be an n-dimensional closed minimally immersed hypersurface in the unit sphere $S^{n+1}(1)$. If the square S of the length of the second fundamental form h on M satisfies $0 \leqslant S \leqslant n$, then $S \equiv 0$ or $S \equiv n$. In [3], S.S. Chern, M. do Carmo and S. Kobayashi proved that the Clifford tori are the only minimal hypersurfaces with $S=n$. C. K. Peng and C. L. Terng [6] studied the case $S=$ constant and shown, among other things, that if $n=3$ and $S>3$, then $S \geqslant 6$. The condition $S=6$ is also assumed in the examples of Cartan [1] and Hsiang [4]. On the other hand, in Otsuki's examples of minimal hypersurface in $S^{n+1}(1)$ (see [5]), H. D. Hu proved that there exist complete and non-compact minimal hypersurfaces in $S^{n+1}(1)$. Hence, it is interesting to study complete minimal hypersurfaces in $S^{n+1}(1)$. In [2], the author considered a compete minimally immersed hypersurface M in $S^{n+1}(1)$ with $S=$constant, and proved that if $0 \leqslant S \leqslant n$, then $S=0$ or $S=n$.

In this paper, we generalize the above theorem due to C. K. Peng and C. L. Terng [6] to complete minimal hypersurfaces. That is, we obtain the following.

Theorem. *Let M^3 be a complete minimally immersed hypersuface in $S^4(1)$ with $S=$constant. If $S>3$, then $S \geqslant 6$.*

Corollary. *Let M^3 be a complete minimally immersed hypersurface in $S^4(1)$ with $S=$constant. If $0 \leqslant S \leqslant 6$, then $S=0$, $S=3$ or $S=6$.*

Proof. According to Theorem and the result of the author [2], Corollary is true obviously.

2. Preliminaries

Let M be an n-dimensional immersed hypersurface in the $n+1$-dimensional unit sphere $S^{n+1}(1)$. We choose a local field of orthonormal frames $e_1, \cdots, e_{n+1}$ in $S^{n+1}(1)$ such that, restricted to M, the vectors $e_1, \cdots, e_n$ are tangent to M. We use the following convention on the range of indices unless otherwise stated: