Oka, M.
Osaka J. Math.
27 (1990), 863–883

SINGULAR FOLIATIONS ON CROSS-SECTIONS OF EXPANSIVE FLOWS ON 3-MANIFOLDS

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(Received September 11, 1989)

1. Introduction

The notion of cross-sections is one of useful methods to investigate the behaviors of flows. H.B. Keynes and M. Sears [6] constructed a family of cross-sections and a first return map for a non-singular flow. In this paper we shall construct singular foliations on cross-sections invariant under the first return maps of flows furnishing expansiveness on three dimensional closed manifolds.

Recently K. Hiraide [5] showed the existence of invariant singular foliations for expansive homeomorphisms of closed surfaces. We shall construct singular foliations on cross-sections by using the method mentioned in [5]. However the first return maps are not continuous and we shall prepare supplementary tools to get our conclusion.

Let X be a closed topological manifold with metric d. By $(X, \boldsymbol{R})$ we denote a real continuous flow (abbrev. flow) without fixed points and the action of $t\in\boldsymbol{R}$ on $x\in X$ is written xt. $(X, \boldsymbol{R})$ is called an *expansive* flow if for any $\varepsilon>0$ there exists $\delta>0$ with the property that if $d(xt, yh(t))<\delta$ $(t\in\boldsymbol{R})$ for a pair of points $x, y\in X$ and for an increasing homeomorphism $h\colon \boldsymbol{R}\to\boldsymbol{R}$ such that $h(0)=0$ and $h(\boldsymbol{R})=\boldsymbol{R}$, then $y=xt$ for some $|t|<\varepsilon$. Every non-trivial expansive flow has no fixed points (see [1]). Hereafter the natural numbers, the integers and the real number will be denoted by $\boldsymbol{N}$, $\boldsymbol{Z}$ and $\boldsymbol{R}$ respectively.

Let $SI=\{xt; x\in S \text{ and } t\in I\}$ for an interval I and $S\subset X$. A subset $S\subset X$ is called a *local cross-section* of time $\zeta>0$ for a flow $(X, \boldsymbol{R})$ if S is closed and $S\cap x[-\zeta, \zeta]=\{x\}$ for all $x\in S$. If S is a local cross-section of time ζ, the action maps $S\times[-\zeta, \zeta]$ homeomorphically onto $S[-\zeta, \zeta]$. By the interior S^* of S we mean $S\cap \operatorname{int}(S[-\zeta, \zeta])$. Note that $S^*(-\varepsilon, \varepsilon)$ is open in X for any $\varepsilon>0$. Put $\varepsilon_0=\inf\{t>0; xt=x \text{ for some } x\in X\}$. Under the above assumptions and notations we have the following

Fact 1.1 ([6], Lemma 2.4). There is $0<\zeta<\varepsilon_0/2$ satisfying that for each $0<\alpha<\zeta/3$ we can find a finite family $\mathcal{S}=\{S_1, S_2, \cdots, S_k\}$ of pairwise disjoint local cross-sections of time ζ and diameter at most α and a family of local corss-