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A NOTE ON PIVOTAL MEASURES IN MAJORIZED EXPERIMENTS

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1. Introduction

An *experiment* $\boldsymbol{E}=(X, \boldsymbol{A}, P)$, i.e. a triplet with a family P of probability measures on a measurable space $(X, \boldsymbol{A})$, is said to be majorized by a measure μ equivalent to P (called a *"majorizing"* measure) if each $p \in P$ has a density $dp/d\mu$ with respect to μ. Let $\boldsymbol{S}_0$ be the σ-ring generated by all the $\boldsymbol{E}$-supports $S(p)=\{x \in X; (dp/d\mu)(x)>0\}, p \in P$. Then there exists a *"maximal decomposition"* $\boldsymbol{F}$ such that $\boldsymbol{F} \subset \boldsymbol{S}_0[P]$ (see [4], Lemma 2). A maximal decomposition $\boldsymbol{F}$ is defined as a covering of X of almost disjoint elements each of which is included by an $\boldsymbol{E}$-support $S(p^{(F)})$ of some $p^{(F)}$ in P. For each $F \in \boldsymbol{F}$, we define a dominated sub-experiment $\boldsymbol{E}(F)=(F, \boldsymbol{A} \cap F, P_F)$ by setting $\boldsymbol{A} \cap F=\{A \cap F; A \in \boldsymbol{A}\}$ and $P_F=\{p_F; p \in P, p(F)>0\}$, where $p_F(A \cap F)=p(A \cap F)/p(F)$. The σ-ring generated by $\{dp/d(p+q) \cdot I_{S(p)}; p, q \in P\}$ is called the σ-ring of *pairwise likelihood ratios* and denoted by $\boldsymbol{S}$. The σ-field $\boldsymbol{D}$ generated by $\boldsymbol{S}$ is known to be the smallest PSS (*pairwise sufficient with supports*) subfield, and is equal to $\boldsymbol{S}$ and minimal sufficient when $\boldsymbol{E}$ is dominated (see [3]). A majorizing measure m on $\boldsymbol{A}$ is said to be *pivotal* for $\boldsymbol{E}$ if it holds that for each subfield $\boldsymbol{B}$, $\boldsymbol{B}$ is PSS if and only if each p has a $\boldsymbol{B}$-measurable version of the density dp/dm. A real valued function f: $X \to \boldsymbol{R}$ is said to be $\boldsymbol{S}$-measurable if for each Borel subset B of $\boldsymbol{R}$, $f^{-1}(B) \cap \{x \in X; f(x) \neq 0\} \in \boldsymbol{S}$.

The notion of pivotal measures was devised to obtain a minimal sufficient subfield by Halmos & Savage [6] and Bahadur [1] for the dominated experiment. Ramamoorthi & Yamada [9] generalized it to the majorized experiment. Recently Luschgy, Mussmann & Yamada [8] proved the following characterization theorem of pivotal measures by the method and in the terminology of vector lattices: Every pivotal measure is represented as the sum of a maximal orthogonal system in the minimal $\boldsymbol{L}$-space.

The *minimal* $\boldsymbol{L}$*-space* is the closed vector sublattice generated by P, and a *maximal orthogonal system* is a family of non-zero measures on $\boldsymbol{A}$ such that any two distinct elements in the family are singular with each other and a measure which is singular with all the elements of the family is zero.