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ON ALMOST RELATIVE PROJECTIVES OVER PERFECT RINGS

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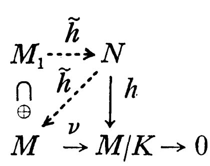
We have defined a new concept of almost relative projectivity [7]. If a module M_0 is M_i-projective for a finite set of modules M_i, then M_0 is $\Sigma_i \oplus M_i$-projective [2]. However this fact is not true for almost relative projectives [7]. We have filled this gap in [6], when a ring R is a semiperfect ring with radical nil and M_0 is a local R-module and the M_i are LE R-modules. As we investigate further several properties of almost relative projectives, it seems for us that the gap is one of essential structures of almost relative projectives. Thus we shall fill completely that gap in this paper, when R is a perfect ring (Main theorem). M_0 was cyclic in [6] and hence the proof was rather simple. Modifying its proof, we shall give a generalization of [6], Theorem 2.

We shall give several applications of the main theorem in forthcoming paper [8], and give the properties dual to this paper in [9].

1. Preliminaries

In this paper we always assume that R is a ring with identity and that every module is a unitary right R-module and e, e' are primitive idempotents unless otherwise stated. We recall here the definition of almost relative projectivity [7]. Let M and N be R-modules. For any diagram with K a submodule of M:



if either there exists $\tilde{h}: N \to M$ with $\nu\tilde{h}=h$ or there exist a nonzero direct summand M_1 of M and $\tilde{h}: M_1 \to N$ with $h\tilde{h}=\nu|M_1$, then N is called *almost M-projective* [7] (if we obtain only the first case, we say that N is *M-projective* [2]).

We note the following fact.

($\sharp$) *When N is almost M-projective and M is indecomposable, if the h in the diagram* (1) *is not an epimorphism, then there exists always an $\tilde{h}: N \to M$ with $\nu\tilde{h}=h$.*