Kambara, H.
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ON DIRECTLY FINITE REGULAR RINGS

Dedicated to Professor Manabu Harada on his sixtieth birthday

HIKOJI KAMBARA

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This paper is concerned with the following open problem for directly finite, von Neuman regular rings. The problem was given by Goodearl and Handelman [3]: what conditions on a regular ring R induce that the maximal right quotient ring of R is right and left self-injective. In [4], the auther showed an example of directly finite, right self-injective regular ring which is not left self-injective. So we have an interest in this problem. In Theorem 17 in §3, we give necessary and sufficient conditions for this problem. In §2, we consider the maximal left quotient ring Q of a directly finite, right self-injective regular ring. We show that Q is directly finite (Theorem 7) and the factor ring $Q/\mathcal{M}$ is the maximal left quotient ring of the factor ring $R/\mathfrak{m}$ for every maximal ideal $\mathcal{M}$ (resp. $\supset\mathfrak{m}$) of Q (resp. R) (Theorem 9). In §3, we give one generalization of a result in [5]: the maximal left quotient ring of a directly finite, right self-injective regular ring is left and right self-injective. Further we obtain necessary and sufficient conditions for the maximal right quotient ring of a regular ring to be directly finite (Theorem 16).

1. Preliminaries

All rings in this paper are associative with unit and ring homomorphisms are assumed to preserve the unit. A ring R is said to be *directly finite* if $xy=1$ implies $yx=1$ for all $x, y\in R$. A ring is said to be *directly infinite* if R is not directly finite. A regular ring means von Neumann regular ring.

A *rank function* on a regular ring R is a map $N: R\rightarrow[0, 1]$ satisfying the following conditions:

(a) $N(1)=1$,

(b) $N(xy)\leqq N(x)$ and $N(xy)\leqq N(y)$ for all $x, y\in R$,

(c) $N(e+f)=N(e)+N(f)$ for all orthogonal idempotents $e, f\in R$,

(d) $N(x)>0$ for all non-zero $x\in R$.

If R is a regular ring with a rank function N, then $\delta(x, y)=N(x-y)$ defines a metric on R, this metric δ is called *N-metric* or *rank metric* and the (Hausdorff)