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LOWER BOUNDS FOR THE MORSE INDEX OF COMPLETE MINIMAL SURFACES IN EUCLIDEAN 3-SPACE

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Introduction

The study of the index of minimal surfaces in Euclidean space has been quite active in these several years. Fischer-Colbrie [7], Gulliver and Lawson [8], [9] have proved independently that a complete oriented minimal surface in $\boldsymbol{R}^3$ has finite index if and only if it has finite total curvature. More recently Tysk [16] has proved that the index of a complete oriented minimal surface in $\boldsymbol{R}^3$ is bounded from above by an explicit constant times the total curvature. For the situation in higher codimensions, see [2], [6] and [13].

In this paper we study the lower bound for the index of complete oriented minimal surfaces in $\boldsymbol{R}^3$. In view of the above mentioned result due to Fischer-Colbrie et al., we may restrict our attention to the surfaces with finite total curvature. It is well known that such a surface is conformally equivalent to a compact Riemann surface with finitely many punctures and the Gauss map of the surface extends to the compactified surface as a holomorphic map. We then give a lower bound for the index in terms of an invariant of the extended Gauss map and the genus of the surface. As a corollary of this result, we give a lower bound for the index in terms of the total curvature of the surface, when all the critical values of the extended Gauss map are contained in some great circle of the target unit sphere. By applying these results we show that the index of the k-end catenoid and the Costa's surface are not less than $2k$-3 and 3 respectively. We also prove that if M is a complete oriented minimal surface of genus zero and is not one of the plane, the Enneper's surface and the catenoid, then the index of M is not less than three. Finally we prove that the index of the k-end catenoid is actually equal to $2k$-3 by explicitly solving the eigenvalue problem associated to the Jacobi operator.

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