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Osaka J. Math.
27 (1990), 421–430

A NOTE ON THE EQUIVARIANT WHITEHEAD GROUPS OF DIHEDRAL GROUPS

Dedicated to Professor Shôrô Araki on his 60th birthday

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(Received March 23, 1989)

0. Introduction

This note is intended as "The equivariant Whitehead torsions of equivariant homotopy equivalence between the unit spheres of representations II". Therefore, we shall use the notations in [11]. In this note, restriction maps in Whitehead groups play an importnat role. To illustrate this, we begin with an example pointed out by M. Masuda. Let C_n and D_n be the cyclic group and dihedral group of order n and $2n$ respectively. As we remarked in [11], a generator of $Wh(C_5)$ appears as the reduced equivariant Whitehead torsion of any C_5-homptopy equivalence

$$f\colon S(V_3 \oplus V_2) \to S(V_1 \oplus V_1).$$

where V_a $(a=1, 2, 3)$ denotes the complex C_5-module $\boldsymbol{C}$ with $g \in C_5$ acting as multiplication by $\exp 2\pi ia/5$ and $S(V)$ denotes the unit sphere of C_5-module V. Since the torsion does not depend on the choice of f, we can assume that f is the map due to T. Petrie (see §2). By the complex conjugation, C_5-modules V_a can be regarded as D_5-modules. Then the Petrie's map f turns out to be a D_5-homotopy equivalence. The reduced equivariant Whitehead torsion $\bar{\tau}_{D_5}(f) = p_* \tau_{D_5}(f)$ of f as a D_5-homotpoy equivalence lies in $Wh_{D_5}(*) \cong Wh(D_5)$ where $p_*\colon Wh_{D_5}(S(V_3 \oplus V_2)) \to Wh_{D_5}(*)$ is the induced map by the obvious map $p\colon S(V_3 \oplus V_2) \to *$. It is obvious that the restriction map from D_5 to C_5 sends the torsion to the generator of $Wh_{C_5}(*) \cong Wh(C_5)$. Therefore the restriction map induces an isomorphism of the Whitehead groups because $Wh(D_5)$ is a free abelian group of rank 1 (see [3], [21], [19], [20] and [17]). Moreover we see that the torsion is a generator of $Wh(D_5)$. Our main result (Theorem A) is a generalization of this observation.

Theorem A. *The restriction map induces an isomorphism*

$$\mathrm{Res}^{D_n}_{C_n}\colon Wh_{\mathrm{rep}}(D_n) \to Wh_{\mathrm{rep}}(C_n),$$