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CHARACTERIZATIONS OF CONDITIONAL EXPECTATION OPERATORS FOR L_p-VALUED FUNCTIONS ON A GENERAL MEASURE SPACE

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Introduction. Let $(\Omega, \boldsymbol{A}, \mu)$ be a measure space, where $\boldsymbol{A}$ is a σ-ring and μ is a σ-finite measure on $\boldsymbol{A}$, (X, S, λ) a measure space and E a real Banach space. We consider semi-constant-preserving contractive projections of $L_1(\Omega, \boldsymbol{A}, \mu, E)$ into itself. If $(\Omega, \boldsymbol{A}, \mu)$ is a probability space and E is a strictly-convex Banach space, then Landers and Rogge [2] proved that such operators coincide precisely with the conditional expectation operators. If $(\Omega, \boldsymbol{A}, \mu)$ is a probability space and $E=L_p(X, S, \lambda)$, where $p=1$ or ∞, then Miyadera [3] and [4] proved that such operators coincide precisely with the conditional expectation operators under some additional conditions. In this paper we deal with the case when $(\Omega, \boldsymbol{A}, \mu)$ is a general measure space, where $\boldsymbol{A}$ is a σ-ring and λ is a σ-finite measure on $\boldsymbol{A}$. Substituting constant-preserving property by semi-constant-preserving property we can prove theorems which are generalizations of characterization theorems in Landers and Rogge [2], Miyadera [3] and [4].

1. Definitions and useful Lemmas. Let $(\Omega, \boldsymbol{A}, \mu)$ be a measure space, $\boldsymbol{A}(\mu)=\{A\in\boldsymbol{A};\ \mu(A)<\infty\}$ and E *a* real Banach space with the norm $\|\ \|$. Note that E can be the class $\boldsymbol{R}$ of real numbers. Let $\boldsymbol{N}$ be the class of natural numbers. For any $A, B\in\boldsymbol{A}$ we write $A\subset B$ if $\mu(A-B)=0$ aud $A=B$ if $\mu((A-B)\cup(B-A))=0$. $A, B\in\boldsymbol{A}$ are said to be disjoint if $\mu(A\cap B)=0$. We suppose that μ is σ-finite, i.e., for any $A\in\boldsymbol{A}$ there exists a sequence of sets $\{A_n;\ n\in\boldsymbol{N}\}$ such that $A_n\in\boldsymbol{A}(\mu)$ and $A=\cup\{A_n;\ n\in\boldsymbol{N}\}$. For any $A\in\boldsymbol{A}$ we denote by I_A the indicator function of A and by $A=\emptyset$ we mean $\mu(A)=0$. Let $L_1(\Omega, \boldsymbol{A}, \mu, E)$ be the calss of E-valued Bochner integrable functions, which is a Banach space with the norm $\|\ \|_L$ defined by

$$\|f\|_L=\int\|f(\omega)\|d\mu \qquad \text{for any} \quad f\in L_1(\Omega, \boldsymbol{A}, \mu, E)\,.$$

For any $f\in L_1(\Omega, \boldsymbol{A}, \mu, E)$ we denote $\{\omega;\ f(\omega)\neq 0\}$ by $s(f)$ and for any linear operator Q of $L_1(\Omega, \boldsymbol{A}, \mu, E)$ into itself we denote $S(Q)=\{A\in\boldsymbol{A}(\mu);$ there