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WEAK CONVERGENCE OF A SEQUENCE OF STOCHASTIC PROCESSES RELATED WITH U-STATISTICS

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1. Introduction

In Hoeffding's fundamental paper [4], he proved the weak convergence of U-statistics under suitable conditions. Loynes [10] and later, Miller and Sen [12] as well as Mandelbaum and Taqqu [11] respectively considered different types of stochastic processes related with U-statistics and studied their weak convergence. In this paper, we are concerned with a sequence of stochastic processes which are similar to those developed by Mandelbaum and Taqqu [11]. We intend to show a deeper analysis of the weak convergence of the processes. This is achieved by using the martingale approach, a method used extensively by Khmaladze [8], [9]. (See Rao [13] for a survey on Martingale approach to Statistical Inference). Under this martingale approach we intend to find natural expressions for the limits of the martingale part and compensator of the processes associated with a sequence of U-statistics. These limits can be expressed by using multiple Wiener integrals.

Let F be a distribution function on $\boldsymbol{R}$ and $X_1, \cdots, X_n$, independent observations on F. Consider a parametric function $\theta=\theta(F)$, for which there exists an unbiased estimator. That is, $\theta(F)$ may be expressed as $\theta(F)=E_F(h(X_1, \cdots, X_m))$ for some function h: $\boldsymbol{R}^m \to \boldsymbol{R}$, called a "kernel", where h can be assumed to be symmetric.

Let's define:

$$h_k(x_1, \cdots, x_k) := E(h(X_1, \cdots, X_m)/X_1 = x_1, \cdots, X_k = x_k) \quad \text{and}$$
$$\zeta_k := Var(h_k(X_1, \cdots, X_k)) \quad \text{for} \quad k = 1, \cdots, m\,,$$

under the assumption

$$E(h^2(X_1, \cdots, X_m)) < \infty\,. \tag{1}$$

The *U-statistic for estimation of θ based on the sample $X_1, \cdots, X_n$ of size* $n \geq m$ is