Hirachi, K.
Osaka J. Math.
27 (1990), 301-308

TRANSFORMATION LAW FOR THE SZEGÖ PROJECTORS ON CR MANIFOLDS

KENGO HIRACHI

(Received April 5, 1989)
(Revised October 6, 1989)

Introduction. Let M be a strictly pseudoconvex CR manifold of dimension $2n+1$. In case a volume element is specified on M, the Szegö projector $S: L^2(M)\rightarrow L^2(M)$ is defined as the orthogonal projector onto the subspace $\ker \bar{\partial}_b$; thus S is not CR-invariant. Assuming that M is the boundary of a strictly pseudoconvex domain $\Omega\subset C^{n+1}$, Fefferman [5] constructed a volume element on M, by using the complex Monge-Ampère operator, in such a way that a natural transformation law for the Szegö projectors holds under CR isomorphisms, cf. (4.1) below. The purpose of this note is to generalize his construction to the case in which M is not necessarily the boundary of a domain.

What we have to do is to seek a right condition on the vloume element on M so as to get the transformation law. Keeping in mind that volume element on M is uniquely determined by contact form, we first specify a family of locally defined contact forms—a step, due to Farris [2], of making Fefferman's construction intrinsic (cf. also Fefferman [4]). As is observed in Farris [2], this is equivalent to giving a family of $(n+1, 0)$-forms on M, closed and nonvanishing. In order to achieve our construction of volume element, it is at first necessary to assume the local existence of a nonvanishing closed $(n+1, 0)$-form in a neighborhood of every point on M. The simplest situation is that there exists a globally defined contact form θ obtained by gluing the $(n+1, 0)$-forms above; if the volume element is given by $\theta\wedge d\theta^n$, then the transformation law for the Szegö projectors (Theorem 1) is derived just as in Fefferman's construction. However, there is a topological obstruction to the global existence of such a contact form. The vanishing of $c(K_M)$, the Chern class of the canonical bundle with real coefficients, is a necessary condition for the global existence. It is not known whether this condition is sufficient (cf. Lee [6] and Remark 1 below); to avoid this difficulty we generalize the notion of the Szegö projector. We construct a complex line bundle, by using the assumption $c(K_M)=0$, *via* transition of the locally defined contact forms in order to define the space of L^2 sections of the bundle, the space on which the Szegö projector is acting; then, the required transformation law (Theorem 2) follows naturally.