Suetake, C.
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ON FINITE POINT TRANSITIVE AFFINE PLANES WITH TWO ORBITS ON l_∞

CHIHIRO SUETAKE

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1. Introduction

Kallaher [3] proposed the following conjecture.

Conjecture. *Let π be a finite affine plane of order n with a collineation group G which is transitive on the affine points of π. If G has two orbits on the line at infinity, then one of the following statements holds:*

(i) *The plane π is a translation plane, and the group G contains the group of translations of π.*
(ii) *The plane π is a dual translation plane, and the group G contains the group of dual translations of π.*

The purpose of this paper is to study this conjecture. When G_A has two orbits of length 1 and n on the line at infinity, where A is an affine point of π, some work has been done on this conjecture. (See Johnson and Kallaher [2].)

Our notation is largely standard and taken from [3]. Let $\mathcal{P}=\pi\cup l_\infty$ be the projective extention of an affine plane π, and G a collineation group of $\mathcal{P}$. If P is a point of $\mathcal{P}$ and l is a line of $\mathcal{P}$, then $G(P, l)$ is the subgroup of G consisting of all perspectivities in G with center P and axis l. If m is a line of $\mathcal{P}$, then $G(m, m)$ is the subgroup consisting of all elations in G with axis m.

In § 2 we prove the following theorem.

Theorem 1. *Let π be a finite affine plane of order n with a collineation group G and let Δ be a subset of l_∞ such that $|\Delta|=t\geq 2$, $(n, t)=1$ and $(n, t-1)=1$. If there is an integer $k_1>1$ such that $|G(P, l_\infty)|=k_1$ for all $P\in\Delta$ and there is an integer $k_2>1$ sucht that $|G(Q, l_\infty)|=k_2$ for all $Q\in l_\infty-\Delta$, then π is a translation plane, and G contains the group T of translations of π.*

In § 3 and § 4, we prove the following theorem by using Theorem 1.

Theorem 2. *Let π be a finite affine plane of order n with a collineation group G which is transitive on the affine points of π. If G has two orbits of length 2 and $n-1$ on l_∞, then one of the following statements holds:*