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THE MAXIMAL QUOTIENT RING OF A LEFT H-RING

Dedicated to Professor Hiroyuki Tachikawa on his sixtieth birthday

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In [2], M. Harada has introduced two new artinian rings which are closely related to QF-rings; one is a left artinian ring whose non-small left module contains a non-zero injective submodule and the other is a left artinian ring whose non-cosmall left module contains a non-zero projective summand. K. Oshiro called the first ring a *left H-ring* and the second one a *left co-H-ring* ([3]). However, later in [5], he showed that a ring R is a left H-ring if and only if it is a right co-H-ring. QF-rings and Nakayama (artinian serial) rings are left and right H-rings ([3]). As the maximal quotient rings of Nakayama rings are Nakayama, it is natural to ask whether the maximal quotient rings of left H-rings are left H-rings. In this note, we show that this problem is affirmative, by determining the structure of the maximal quotient rings of left H-rings.

1. Preliminaries

Throughout this paper, we assume that all rings R considered are associative rings with identity and all R-modules are unital. Let M be a R-module. We use $J(M)$ and $S(M)$ to denote its Jacobson radical and its socle, respectively.

Definition [3]. A module is *non-small* if it is not a small submodule of its injective hull. We say that a ring R is a *left H-ring* if R is a left artinian ring satisfying the condition that every non-small left R-module contains a non-zero injective submodule.

We note that a left H-ring is also right artinian by [7, Th. 3]. In [5], for a left H-ring R, K. Oshiro gave the following theorem, by using M. Harada's results of [2, Th. 3.6.]: a ring R is a left H-ring if and only if it is left artinian and its complete set E of orthogonal primitive idempotents is arranged as $E=\{e_{11}, \cdots, e_{1n(1)}, \cdots, e_{m1}, \cdots, e_{mn(m)}\}$ for which

(1) each $e_{i1}R$ is injective,

(2) for each i, $e_{ik-1}R\cong e_{ik}R$ or $J(e_{ik-1}R)\cong e_{ik}R$ for $k=2, \cdots, n(i)$, and

(3) $e_{ik}R\not\cong e_{jt}R$ if $i\neq j$.