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ON THE SMALLEST PAIRWISE SUFFICIENT SUBFIELD IN THE MAJORIZED STATISTICAL EXPERIMENT

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1. Introduction

In the present paper, we discuss the question of the existence of the smallest pariwise sufficient subfield in majorized statistical experiments.

Let $\mathcal{E}=(X, \mathcal{A}, \mathcal{P})$ be a *statistical experiment,* i.e. X be a set, $\mathcal{A}$ a σ-field of subsets of X and $\mathcal{P}$ a family of probability measures on $\mathcal{A}$.

Assume, throughout the present paper, that there exists a "*majorizing*" measure μ on $\mathcal{A}$, with respect to which each P in $\mathcal{P}$ has an $\mathcal{A}$-measurable density $dP/d\mu$. Accordingly, $\mathcal{E}$ is called a *majorized experiment.*

For each $P\in\mathcal{P}$, $S_P=\{x\in X; dP/d\mu(x)>0\}$ is called an *$\mathcal{E}$-support* of P. We notice that S_P is uniquely determined up to a $\mathcal{P}$-null set and satisfies (1) $P(S_P)=1$, and (2) if $N\subset S_P$ and $P(N)=0$, then N is $\mathcal{P}$-null (see section 2). Conversely, if each P has an $S_P\in\mathcal{A}$ satisfying (1) and (2), then, not only $\mathcal{E}$ is majorized, but it has an "*equivalent majorizing measure*" ν, that is, all the $\mathcal{P}$-null sets are ν-null (see [4] Lemma 9.3). Consequently, every majorized experiment has an equivalent majorizing measure.

A sub σ-field $\mathcal{B}$(or simply a subfield) of $\mathcal{A}$, ahich is pairwise sufficient and contains a version of the support S_P for all P in $\mathcal{P}$ is called PSS (*pairwise sufficient with supports*). This is a concept in between the usual concepts of sufficiency and pairwise sufficiency. All the three concepts coincide with each other in case $\mathcal{E}$ is dominated. In each of the classes of the pairwise sufficient, PSS and the sufficient subfields, the smallest and the minimal subfields are defined as follows.

For two subfields $\mathcal{B}$, $\mathcal{C}$ of $\mathcal{A}$, we write $\mathcal{B}\subset\mathcal{C}[\mathcal{P}]$ if $\mathcal{B}\subset\mathcal{C}\vee\mathcal{N}_{\mathcal{P}}$, the latter being the subfield generated by $\mathcal{C}$ and all the $\mathcal{P}$-null sets. If $\mathcal{B}\subset\mathcal{C}[\mathcal{P}]$ and $\mathcal{C}\subset\mathcal{B}[\mathcal{P}]$, we write $\mathcal{B}=\mathcal{C}[\mathcal{P}]$.

A pairwise sufficient (resp. PSS, sufficient) subfield $\mathcal{B}$ is called *smallest* if $\mathcal{B}\subset\mathcal{C}[\mathcal{P}]$ for every pairwise sufficient (resp. PSS, sufficient) subfield $\mathcal{C}$. A pairwise sufficient (resp. PSS, sufficient) subfield $\mathcal{B}$ is called *minimal* if for every pairwise sufficient (resp. PSS, sufficient) subfield $\mathcal{C}$ with $\mathcal{B}\subset\mathcal{C}[\mathcal{P}]$, it holds that $\mathcal{B}=\mathcal{C}[\mathcal{P}]$.