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ORIENTATION REVERSING INVOLUTIONS ON CLOSED 3-MANIFOLDS

Dedicated to Professor Fujitsugu Hosokawa on his 60th birthday

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1. Introduction.

Let M be a closed connected oientable 3-manifold admitting an oreintation reversing involtuion τ (i.e. $\tau^2=$identity and $\tau_*([M])=-[M]$ for the fundamental class $[M]$ of M).

By Smith theory, each component of the fixed point set of τ, Fix(τ, M), is a point or a closed surface and $\chi(\text{Fix}(\tau, M))\equiv 0 \pmod 2$ ($\chi(X)$ is the Euler characteristic of X). A. Kawauchi [5] proved that for any (M, τ), Tor $H_1(M; \boldsymbol{Z}) \cong A\oplus A$ or $\boldsymbol{Z}_2\oplus A\oplus A$ for some abelian group A, and that $\text{rank}_{\boldsymbol{Z}_2} H_1(\text{Fix}(\tau, M); \boldsymbol{Z}_2)\equiv 0 \pmod 2$ if and only if Tor $H_1(M; \boldsymbol{Z})\cong A\oplus A$. J. Hempel has proved in [3] that if Fix(τ, M) is empty or contains a closed orientable surface of positive genus, then the first Betti number of M is greater than zero. He has also shown in [4] that if $\pi_1(M)$ is not isomorphic to $\{1\}$ or and $\boldsymbol{Z}_2$ is not virtually representable to $\boldsymbol{Z}$, then Fix(τ, M) consists of a 2-sphere or two points, or contains a projective plane.

The auther gave a characterization of Fix(τ, M) when M is a rational homoogy 3-sphere in [6] and, for a general M, an inequality on the first Betti numbers of M and Fix(τ, M) in [7]. In this paper we give a complete characterization of the topological type of Fix(τ, M) for a general M.

NOTATIONS. For a space X, let $\beta_i(X)$ denote the i^{th} Betti number and $\beta_i(X; \boldsymbol{Z}_2)$ the $\boldsymbol{Z}_2$-coefficient Betti number. For a group G, let $\beta_1(G)=\text{rank}_{\boldsymbol{Z}} H_1(G; \boldsymbol{Z})$ and $\beta_1(G; \boldsymbol{Z}_2)=\text{rank}_{\boldsymbol{Z}_2} H_1(G; \boldsymbol{Z}_2)$.

First, we classify (M, τ) into two types.

Proposition 1. *For any (M, τ), one of the following holds:*
(1) $M-\text{Fix}(\tau, M)$ *consists of two components and* Fix(τ, M) *is a closed orientable 2-manifold.*
(2) $M-\text{Fix}(\tau, M)$ *is connected.*

For each type of (M, τ), we shall prove the following: