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SEMIFIELD PLANES OF CHARACTERISTIC p ADMITTING p-PRIMITIVE BAER COLLINEATIONS

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1. Introduction

Let π denote a semifield plane of order q^2 and kernel $K\cong GF(q)$ where q is a prime power p^r. We shall say that π admits a *p-primitive Baer collineation* σ if and only if σ is a collineation which fixes a Baer subplane pointwise and $|\sigma|$ is a p-primitive divisor of $q-1$ (i.e. $|\sigma|\,|q-1$ but $\not| p^i-1$ for $1\leq i<r$).

The main result of this article is essentially that p-primitive Baer collineations are easy to come by and the class of such semifield planes characterize *all* dimension two semifield planes.

Theorem 2.1. *The class of semifield planes of dimension two and characteristic p which admit a p-primitive Baer collineation is equivalent to the general class of semifield planes of dimension two and characteristic p (either class constructs the other).*

NOTATION 1.1. Let π be any semifield plane of order q^2 and kernel $\geqq K\cong GF(q)$ (π could be Desarguesian). Represent $\pi=\{(x_1, x_2, y_1, y_2)\,|\,x_i, y_i\in K, i=1, 2\}$, $x=(x_1, x_2)$, $y=(y_1, y_2)$, $\mathcal{O}=(0, 0)$. If $x=\mathcal{O}$ is a shears axis then the spread for π takes the following form $x=\mathcal{O}, y=x\begin{bmatrix}\alpha, & \beta\\ \bar{g}(\alpha, \beta), & h(\alpha, \beta)\end{bmatrix}$ where $\bar{g}$, h are biadditive maps (a function f: $K\times K\rightarrow K$ is biadditive$\Leftrightarrow f(\alpha, \beta)+f(\delta, \gamma)=f(\alpha+\delta, \beta+\gamma)$).

(1.2) Extensions of π. (See Hiramine et al. [2] and Johnson [3])

With the notation of (1.1), extend K by t so that $K[t]\cong GF(q^2)$ and $t^2=t\theta+\rho$ for $\theta, \rho\in K$.

Define $g(\alpha, \beta)=\bar{g}(\alpha, \beta)+\theta h(\alpha, \beta)$. Further define $f(\alpha+\beta t)=g(\alpha, \beta)-h(\alpha, \beta)t$ for all $\alpha, \beta\in K$. Then (Johnson [3] (3.1)) $x=\mathcal{O}, y=x\begin{bmatrix}\alpha, & \beta\\ \bar{g}(\alpha, \beta), & h(\alpha, \beta)\end{bmatrix}$ represents the spread for π if and only if

$$x=\mathcal{O},\ y=x\begin{bmatrix}\delta+\gamma t, & \alpha+\beta t\\ g(\alpha, \beta)-h(\alpha, \beta)t, & (\delta+\gamma t)^q\end{bmatrix}=\begin{bmatrix}u, & v\\ f(v), & u^q\end{bmatrix},$$