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ON DOMINANT DIMENSION OF NOETHERIAN RINGS

Dedicated to Professor Hiroyuki Tachikawa on his 60th birthday

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Throughout this note, R stands for a ring with identity and all modules are unital modules. In this note, for a given module M, we say that M has *dominant dimension* at least n, written dom dim $M \geqq n$, if each of the first n terms of the minimal injective resolution of M is flat. Following Morita [5], we call R left (resp. right) QF-3 if dom dim ${}_RR \geqq 1$ (resp. dom dim $R_R \geqq 1$). He showed that if R is left noetherian and left QF-3 then it is also right QF-3. Thus, if R is left and right noetherian, R is left QF-3 if and only if it is right QF-3. Generalizing this, we will prove the following

Theorem. *Let R be left and right noetherian. For any $n \geqq 1$,* dom dim ${}_RR \geqq n$ *if and only if* dom dim $R_R \geqq n$.

In case R is artinian, our dominant dimension coincides with Tachikawa's one [8], and the above theorem has been established (see Tachikawa [9] for details).

In what follows, for a given left or right R-module M, we denote by M^* the R-dual of M, by $\varepsilon_M: M \to M^{**}$ the usual evaluation map and by $E(M)$ the injective hull of M. We denote by mod R (resp. mod R^{op}) the category of all finitely generated left (resp. right) R-modules, where R^{op} stands for the opposite ring of R and right R-modules are considered as left R^{op}-modules.

1. Preliminaries. In this section, we recall several known facts which we need in later sections.

Lemma 1.1. *Let R be right noetherian. For any $N \in$ mod R^{op} and for any injective left R-module E,* $\mathrm{Hom}_R(\mathrm{Ext}^i_R(N, R), E) \simeq \mathrm{Tor}^R_i(N, E)$ *for $i \geqq 1$.*

Proof. See Cartan and Eilenberg [1, Chap. VI, Proposition 5.3].

Lemma 1.2. *Every finitely presented submodule of a flat module is torsionless.*

Proof. See Lazard [4, Théorème 1.2].

Lemma 1.3. *Let R be right noetherian. Let E be an injective left R-module*