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ON CHARACTER CORRESPONDENCES IN π-SEPARABLE GROUPS

Dedicated to Professor Tuyosi Oyama on his 60th birthday

MASAKI TANAKA

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1. Introduction

Let A and G be finite groups and suppose A acts on G by automorphisms. We denote by $\mathrm{Irr}(G)$ the set of ordinary (complex) irreducible characters of G. For a prime p, $\mathrm{IBr}_p(G)$ denotes the set of all irreducible Brauer characters of G with respect to p. If φ is a class function of G and $a \in A$, φ^a, defined by $\varphi^a(g^a)=\varphi(g)$ for $g \in G$, is again a class function. For a set S of class functions of G which is stable under the action of A, we write S_A to denote the set of all A-invariant elements of S. Let π be a set of prime numbers and let π' be the set of primes complementary to π. For $\chi \in \mathrm{Irr}(G)$, we denote by $\hat{\chi}$ the restriction of χ to the set $\hat{G}$ of all π-elements of G. If $\hat{\chi}$ can not be written in the form $\hat{\chi}=\hat{\zeta}+\hat{\psi}$ with ordinary characters ζ, ψ of G, then we say that χ is π-irreducible and that $\hat{\chi}$ is a π-irreducible character of G. We denote the set of all π-irreducible characters of G by $\mathrm{I}_\pi(G)$. We say that G is π-separable if every composition factor of G is either a π-group or a π'-group.

For a π-separable group G, Isaacs [8] considered the vector space c.f. $(\hat{G})$ of all complex-valued class functions defined on $\hat{G}$ and showed that $\mathrm{I}_\pi(G)$ is a basis of c.f. $(\hat{G})$ which has the following properties.

(1) If $\chi \in \mathrm{Irr}(G)$, then $\hat{\chi}$ is a nonnegative integer linear combination of elements of $\mathrm{I}_\pi(G)$.

(2) If $\varphi \in \mathrm{I}_\pi(G)$, then $\varphi=\hat{\chi}$ for some $\chi \in \mathrm{Irr}(G)$. These imply that $\mathrm{I}_\pi(G)$ behaves as a π-generalization of Brauer characters.

Now assume that A acts on G by automorphisms and $(|A|, |G|)=1$. Under the assumption that A is solvable, Glauberman [2] established a natural bijection from $\mathrm{Irr}(G)_A$ onto $\mathrm{Irr}(C_G(A))$. If A is non-solvable, then $|A|$ is even by the Odd-order Theorem and hence $|G|$ is odd. In that case, Isaacs [4] showed that there also exists a similar bijection from $\mathrm{Irr}(G)_A$ onto $\mathrm{Irr}(C_G(A))$.

On the other hand, Uno [10] studied a character correspondence between Brauer characters. He proved that if G is p-solvable, then there exists a bijection from $\mathrm{IBr}_p(G)_A$ onto $\mathrm{IBr}_p(C_G(A))$ and this has similar properties as those of