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CONTINUITY ESTIMATES FOR SOLUTIONS OF PARABOLIC EQUATIONS ASSOCIATED WITH JUMP TYPE DIRICHLET FORMS

TAKASHI KOMATSU

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0. Introduction

A priori estimates for solutions of linear equations generally play an important part in the study of non-linear equations. It is important that the estimates do not depend at all on the smoothness of coefficients. For uniformly parabolic equations of the form

$$\frac{\partial u}{\partial t} = \Sigma_{ij}\, \partial_i(a_{ij}(t, x)\partial_j u)\,, \tag{0.1}$$

the pioneering result of this kind is the Hölder continuity estimate due to Nash [14]. Generalizations of Nash's theorem were obtained by various methods in a number of works (see, for example, [8], [12]).

In the appendix to [14], Nash stated without detailed proof a Harnack type inequality for solutions of (0.1), while Moser [13] gave a proof of the Harnack type inequality different from that by Nash, and obtained the Hölder continuity estimate using the Harnack type inequality. For equation (0.1) with discontinuous coefficients, Aronson [1] proved the uniqueness of weak solutions of the Cauchy problem making use of Moser's result. The Harnack type inequality for solutions of quasi-linear parabolic equations was discussed in Aronson-Serrin [4] following the outline of Moser's proof. Applying theorems in [4], Aronson [2] showed that the fundamental solution of equation (0.1) is bounded below and above by functions of the form

$$Ct^{-d/2} \exp[-\beta|x|^2/t]\,,$$

where d is the dimension of the space, C and β denote positive constants. (cf. Ladyzhenskaja-Solonnikov-Uraltseva [11], Aronson [3]).

Parabolic equations of second order are closely related to Markov processes, so that Nash's methods have some probabilistic flavour. Above mentioned works were applied to various problems in stochastic analysis, especially to stochastic optimal control (cf. Bensoussan-Lions [5]) and homogenization (cf. Fukushima