Sugita, H.
Osaka J. Math.
25 (1988), 665–696

POSITIVE GENERALIZED WIENER FUNCTIONS AND POTENTIAL THEORY OVER ABSTRACT WIENER SPACES

HIROSHI SUGITA[1]

(Received May 8, 1987)

Introduction

The notion of generalized Wiener functions (or "functionals" if the underlying space is a function space) has been introduced in a development of the Malliavin calculus ([17] [20] [21]). This is a natural infinite dimensional analogy of the Schwartz distribution theory in which the role of the Lebesgue measure on a Euclidean space $\boldsymbol{R}^n$ is now replaced by a Gaussian measure on a Banach space. (Such a measure space is called an abstract Wiener space.) In this paper, we will show that generalized Wiener functions which are positive, i.e., those which yield non-negative values when they act upon positive test functions, are measures on the underlying Banach space. It is an analogue of the fact that positive Schwartz distributions are measures.

The class of measures corresponding to positive generalized Wiener functions contains many measures which are singular with respect to the original Gaussian measure and yet, in contrast with finite dimensional cases, it constitutes a rather small class in the totality of finite Borel measures on the Banach space. Many properties of this class can be stated in terms of the potential theory over the abstract Wiener space, particularly, in terms of *capacities*. Such a potential theory has been discussed, among others, by Malliavin [10], Fukushima [3], Fukushima-Kaneko [4] and Takeda [19]. We will show in this paper that these measures can not have their mass in any set of capacities zero and that, on the contrary, for any set of non-zero capacity, there exists a non-trivial measure in this class which is supported on the closure of the set.

In many probelms of extending results in finite dimensional spaces to those in infinite dimensional spaces, a difficulty usually occurs from the fact that an infinite dimensional vector space is *not* locally compact. Indeed, this is the case in our problem, too. However, this difficulty can be fortunately overcome by the fact that a probability measure on a complete separable metric space is

---

1) Supported partially by the YUKAWA foundation.